Indexing the Bijective BWT

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FM Index

ingredients

- BWT
- wavelet tree

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operation: backward search

- locate pattern
- time independent on number of occurrences
- O(|*P*|) rank/select for pattern *P*

FM Index

on bijective BWT

ingredients

- bijective BWT
- wavelet tree

operation: backward search

- locate pattern
- time independent on number of occurrences

$O(|P| \lg |P|)$ rank/select for pattern P

bijective BWT is
the BWT of
the Lyndon factorization
of an input text
with respect to $\prec_\omega$

bijective BWT is
the BWT of
the Lyndon factorization 1.
of an input text
with respect to $\prec_\omega$ 2.

Lyndon words

- a
- aabab

Lyndon word is smaller than
- any proper suffix
- any rotation

Lyndon words

- a
- aabab

Lyndon word is smaller than
- any proper suffix
- any rotation

not Lyndon words:

- abaab (rotation aabab smaller)
- abab (abab not smaller than suffix ab)

Lyndon factorization [Chen+ '58]

- input: text T
- output: factorization $T_1 ... T_t$ with
  - T_i is Lyndon word
  - $T_x \geq_{\text{lex}} T_{x+1}$
  - factorization uniquely defined
  - linear time [Duval'88]

properties [Duval' 88]

- T_t:
  - smallest Lyndon word
  - smallest suffix of T
- T_x primitive
- T_1 longest Lyndon prefix of T[1..]
- T_{x+1} longest Lyndon prefix of $T[|T_1 \cdots T_x|+1..]$

$\prec_\omega$

- $u \prec_\omega w :\Longleftrightarrow uuuu\ldots <_{\text{lex}} wwww\ldots$

- $\mathtt{ab} <_{\text{lex}} \mathtt{aba}$
- $\mathtt{aba} \prec_\omega \mathtt{ab}$

$\prec_\omega$

- $u \prec_\omega w :\Longleftrightarrow uuuu\ldots <_{\text{lex}} wwww\ldots$

- $ab <_{\text{lex}} aba$
- $aba \prec_\omega ab$

abababab⋯
ababaaba⋯

bijective BWT of senescence

s|enes|cen|ce

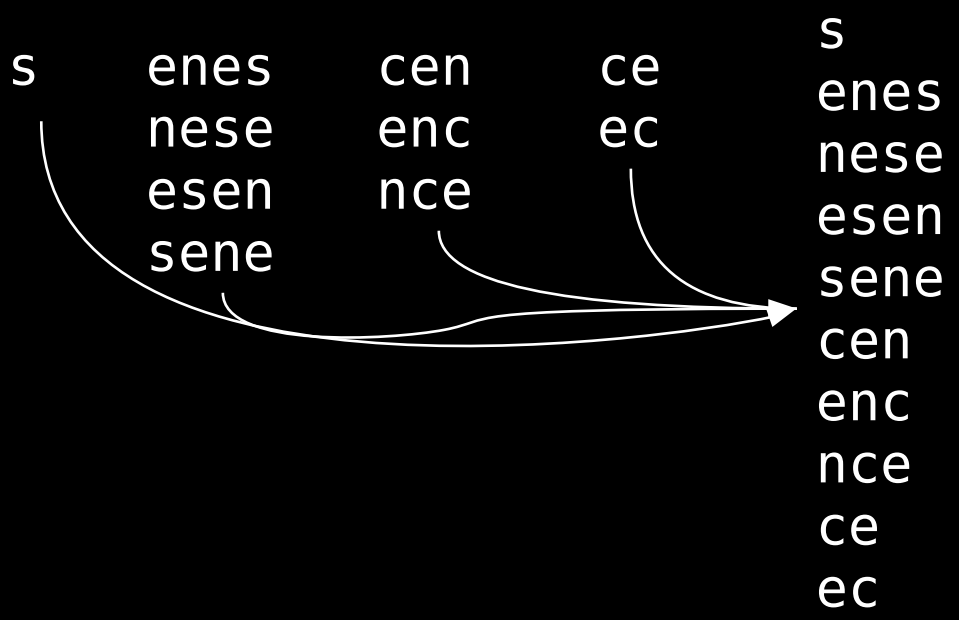
bijective BWT of senescence

s|enes|cen|ce

```
s    enes    cen    ce
     nese    enc    ec
     esen    nce
     sene
```

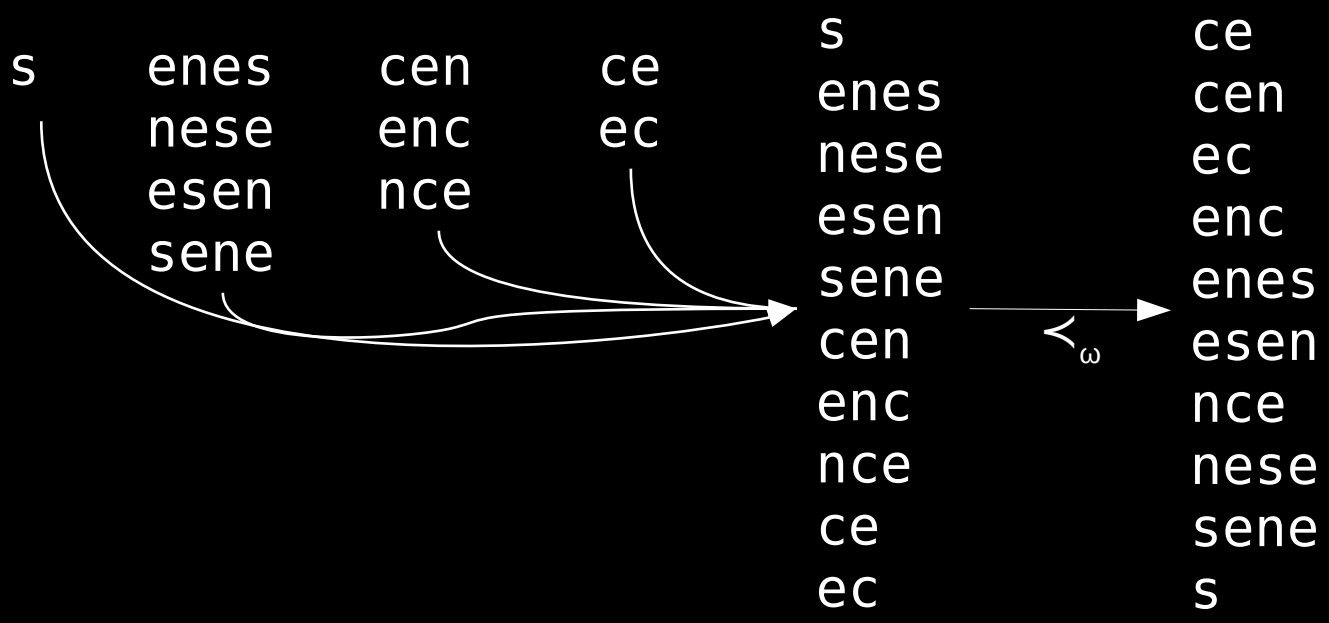
bijective BWT of senescence

s|enes|cen|ce



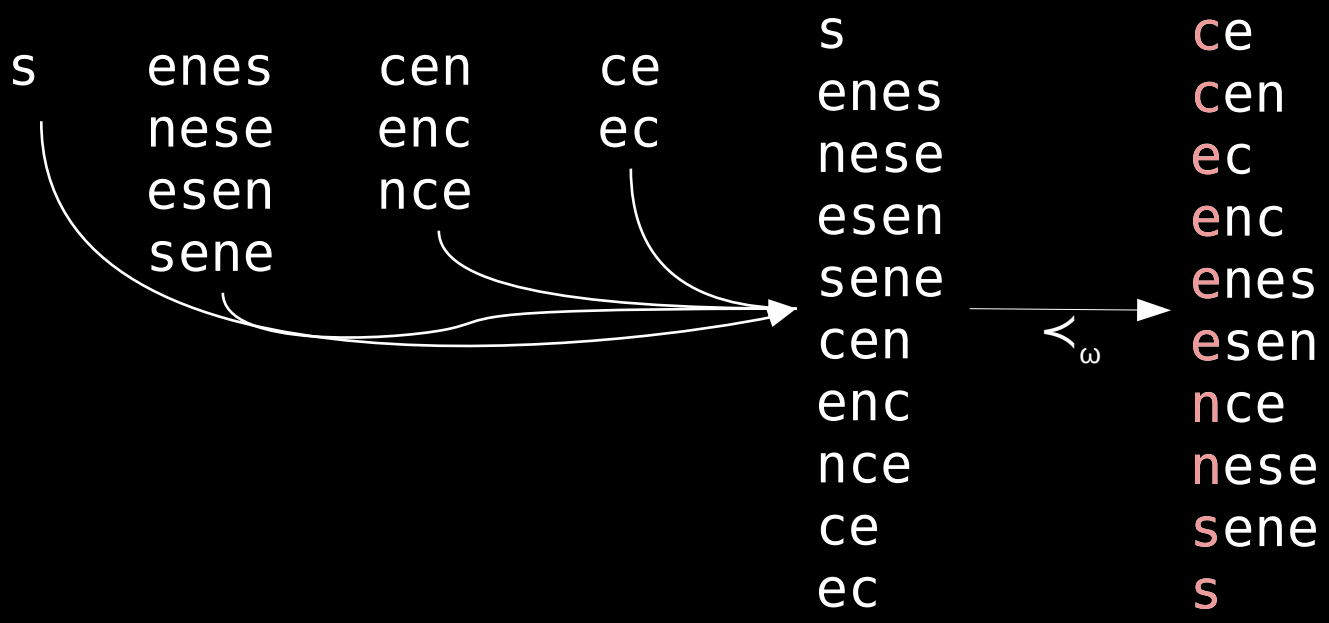
bijective BWT of senescence

s|enes|cen|ce



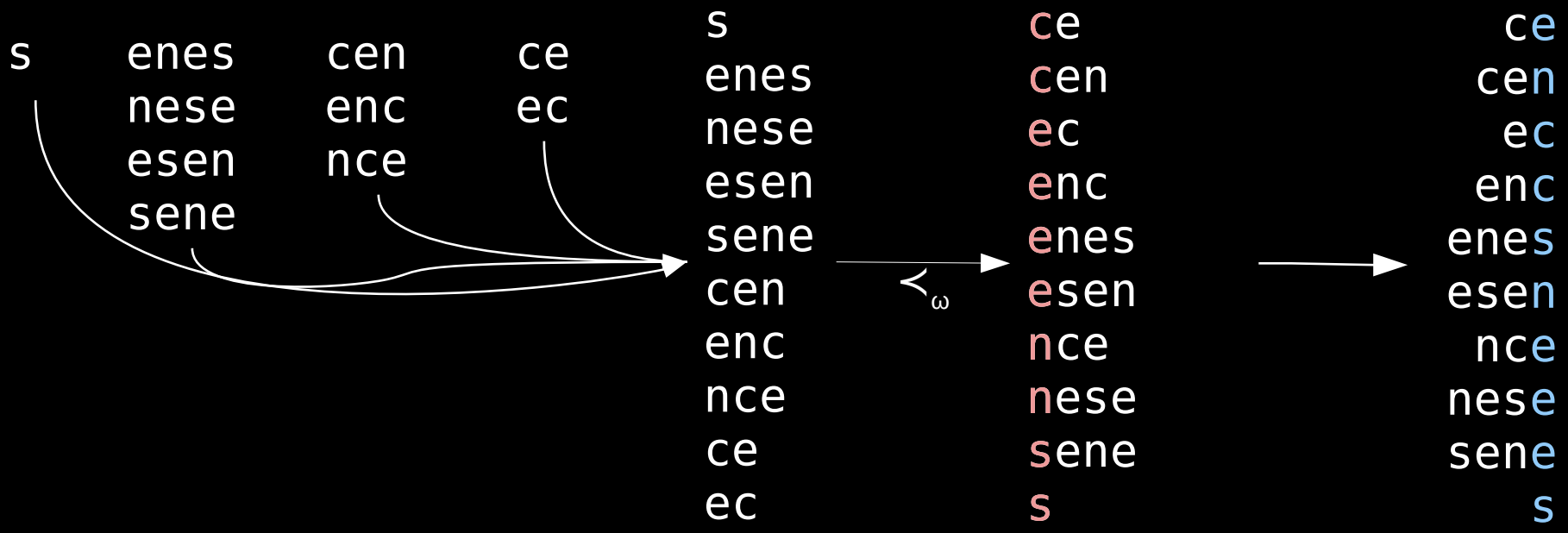
bijective BWT of senescence

s|enes|cen|ce



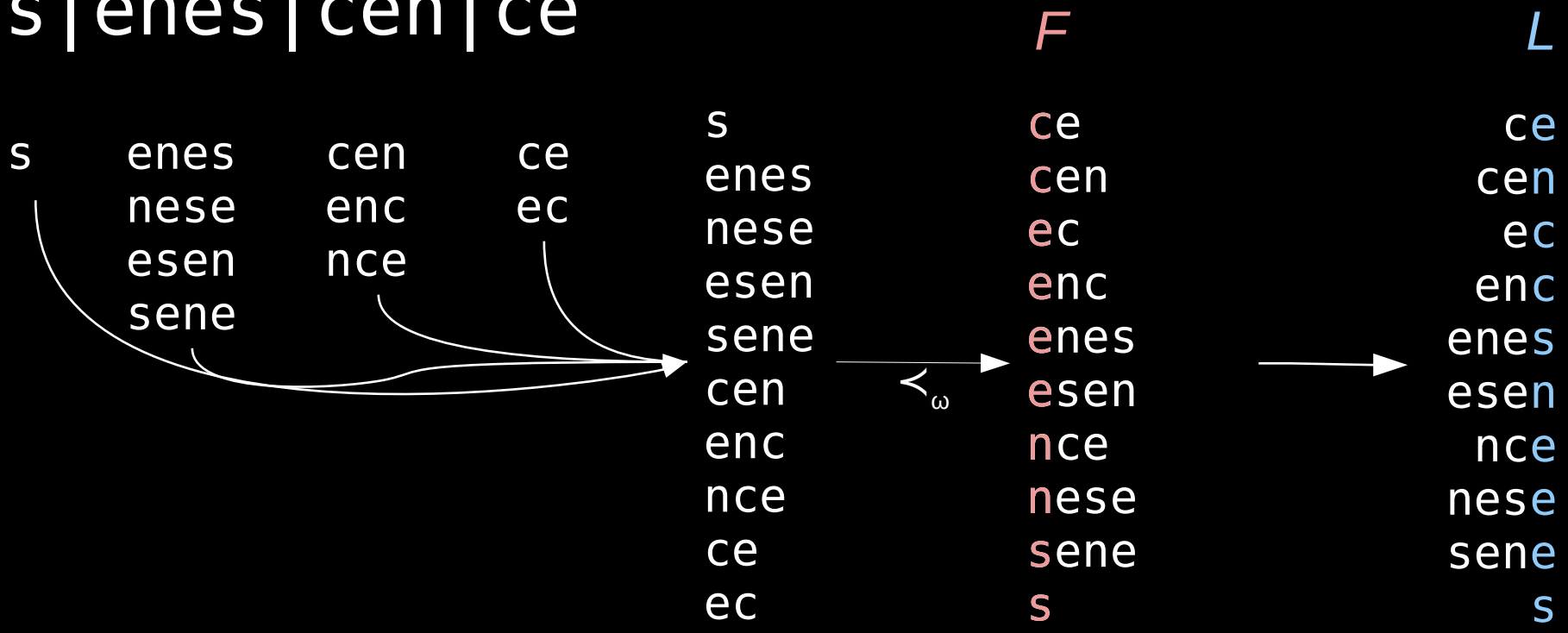
bijective BWT of senescence

s|enes|cen|ce



bijective BWT of senescence

s|enes|cen|ce

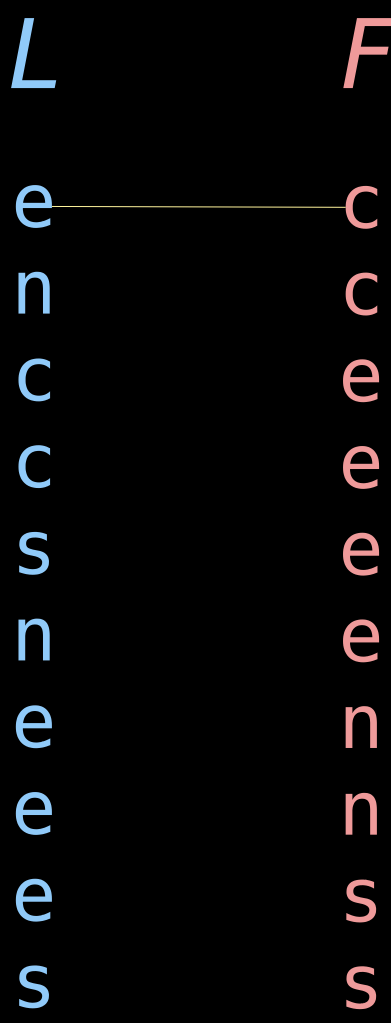


result: enccsneees

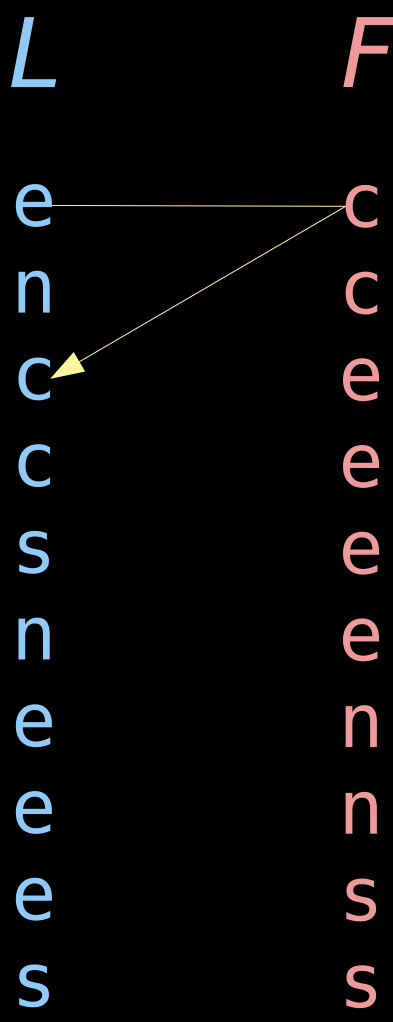
cycles

L	*F*
e	c
n	c
c	e
c	e
s	e
n	e
e	n
e	n
e	s
s	s

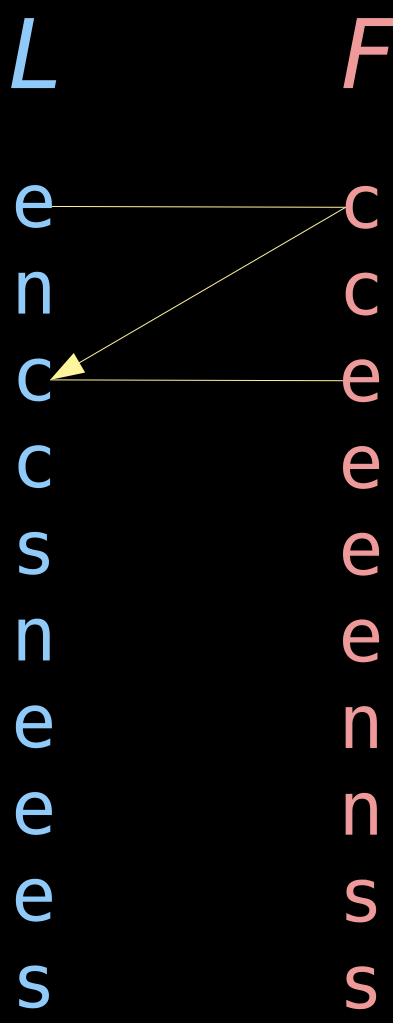
cycles



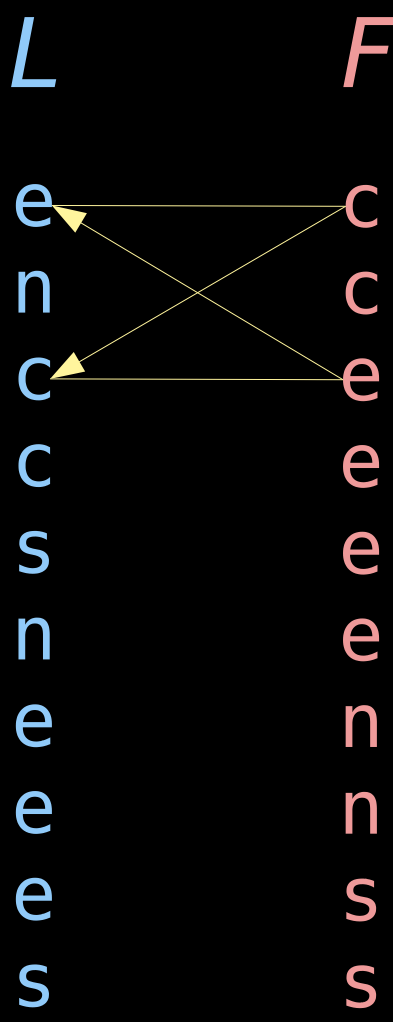
cycles



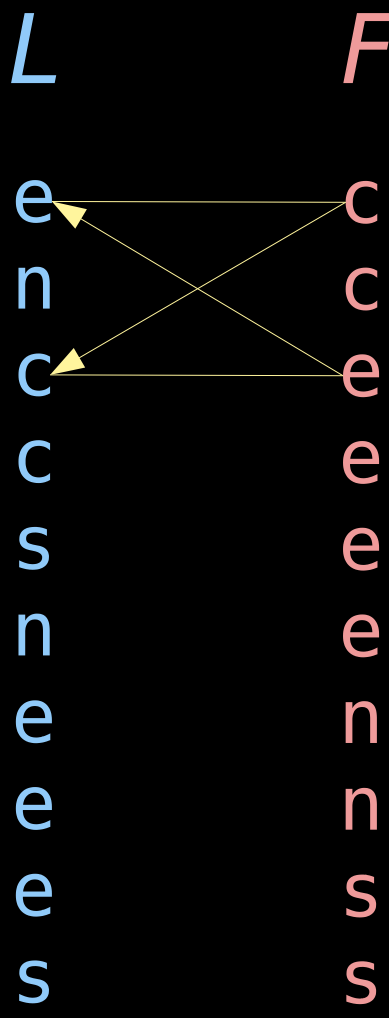
cycles



cycles

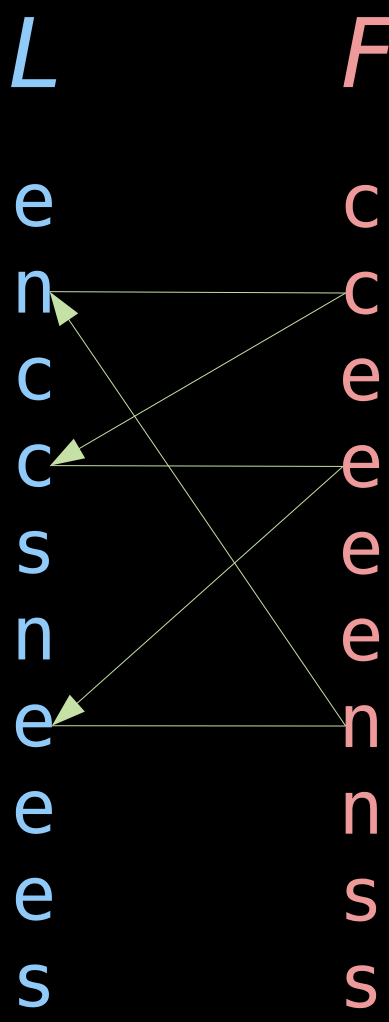


cycles



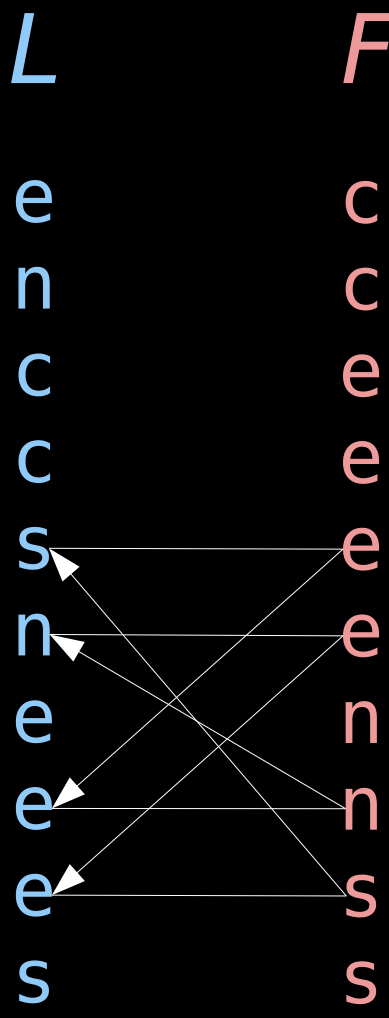
ce

cycles



ce

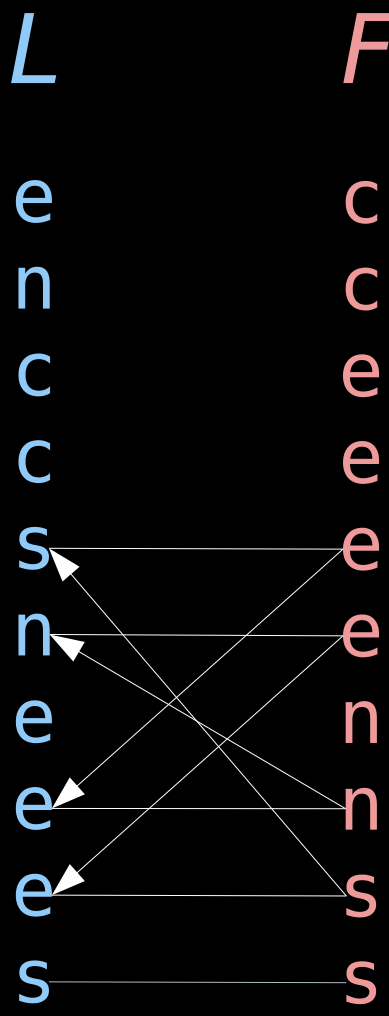
cycles



ce

enes

cycles

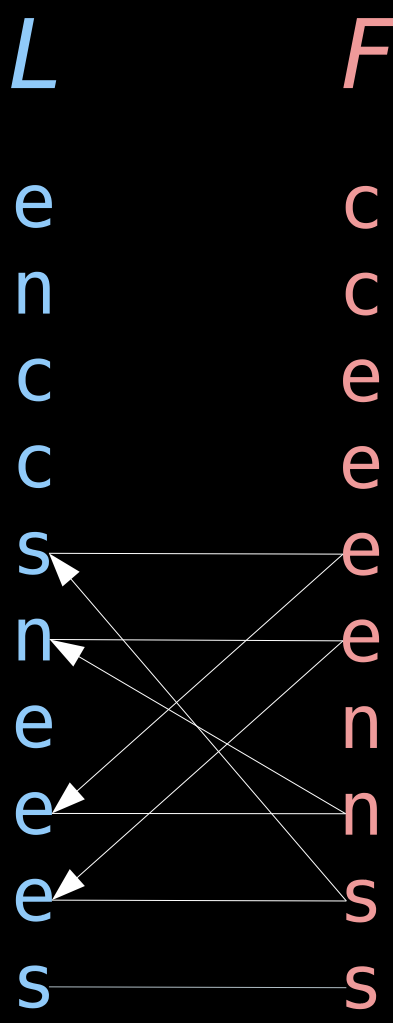


ce

enes

s

cycles


L
F
e
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c
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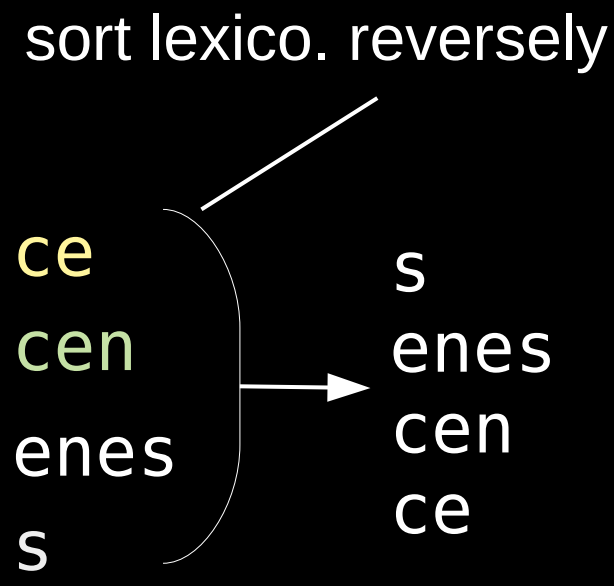
s
n
e
e
e
s
c
c
e
e
e
n
n
s
s


sort lexico. reversely
ce
cen
enes
s
s
enes
cen
ce

backward search 'cen'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

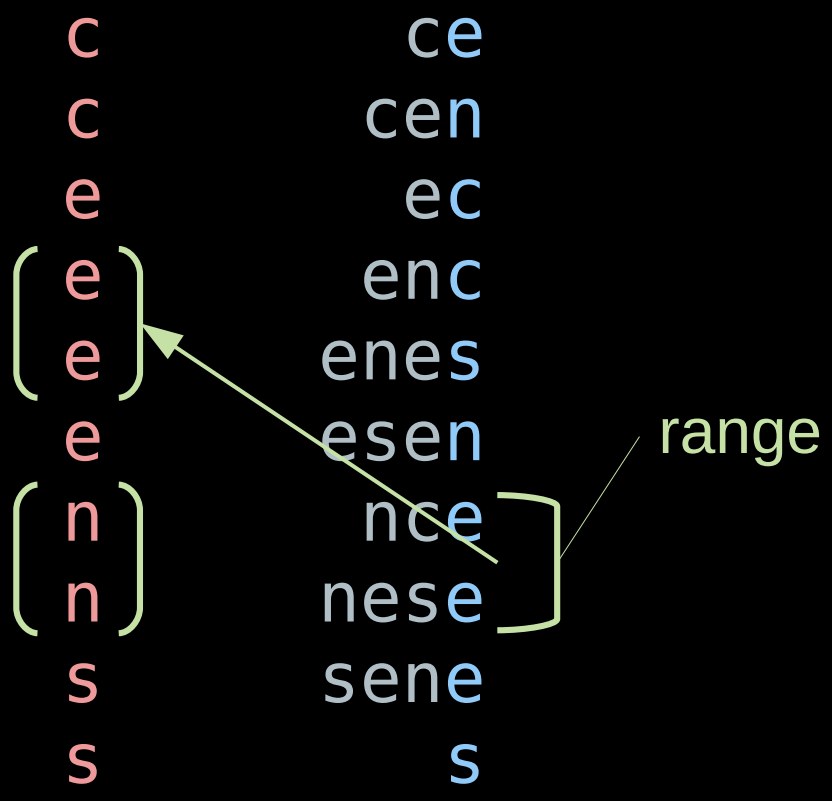
backward search 'cen'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

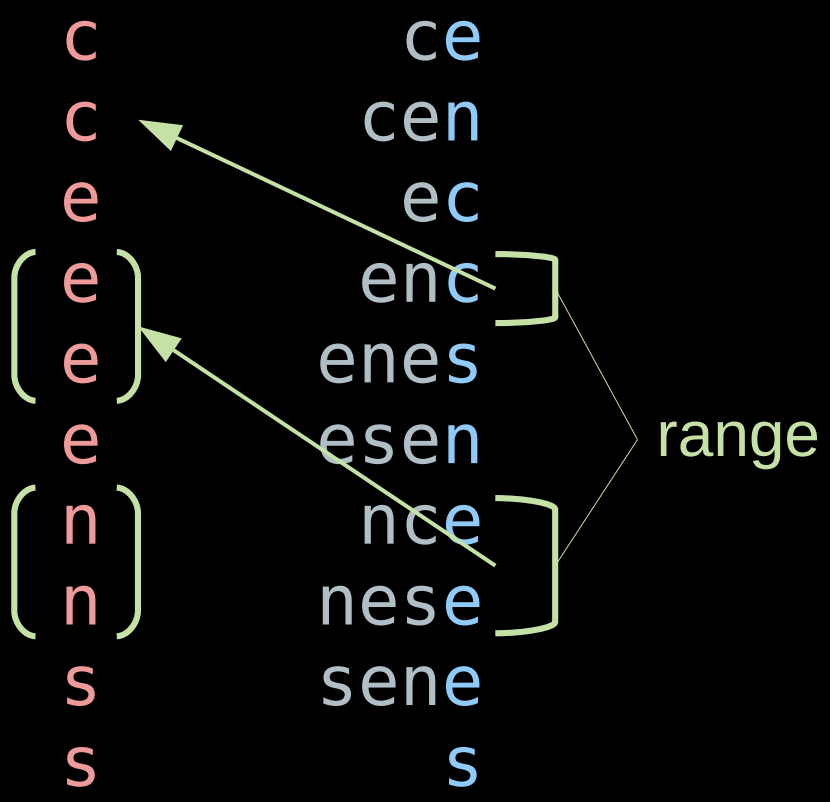
backward search 'cen'

s|enes|cen|ce

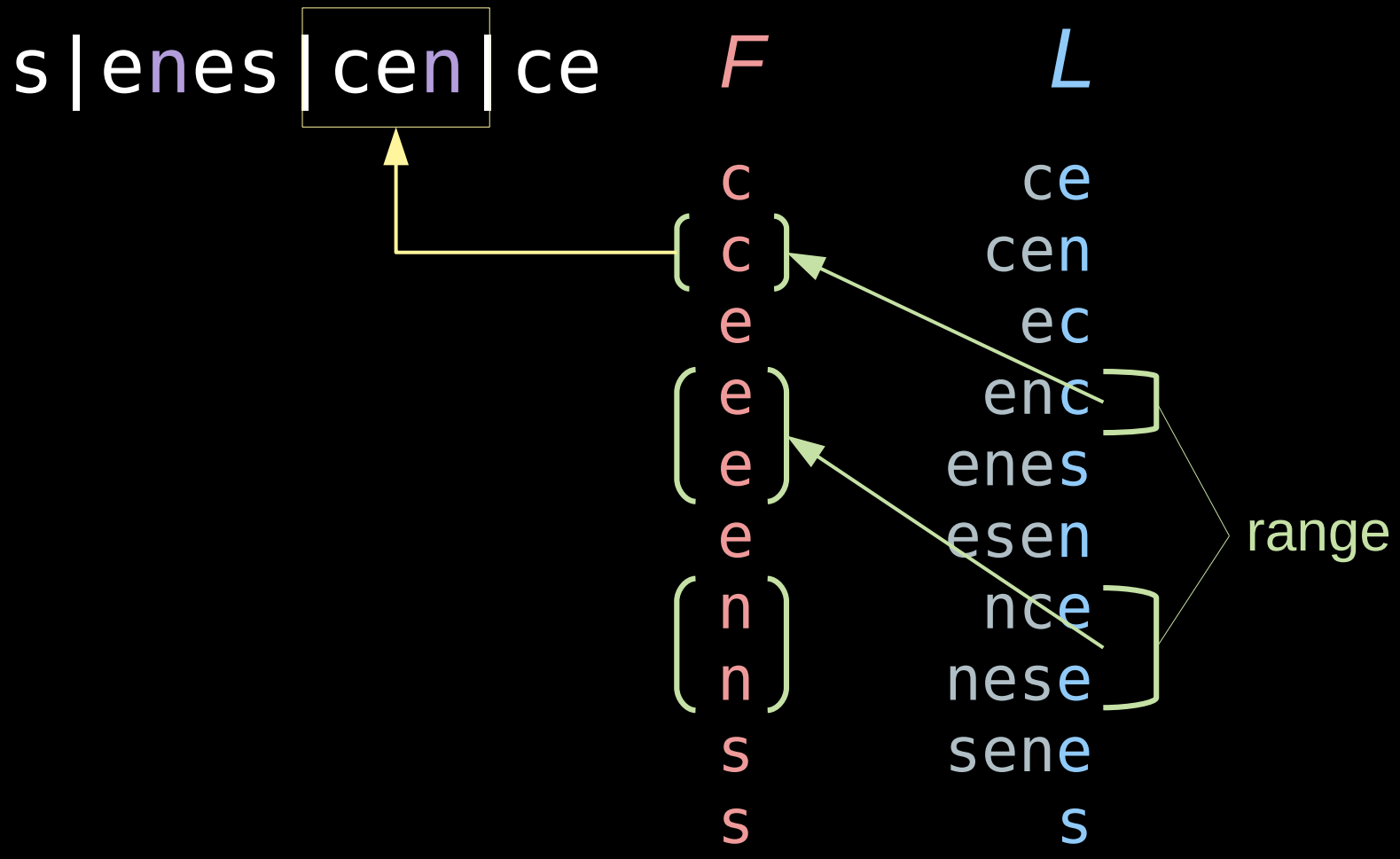


backward search 'cen'

s|enes|cen|ce



backward search 'cen'



backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

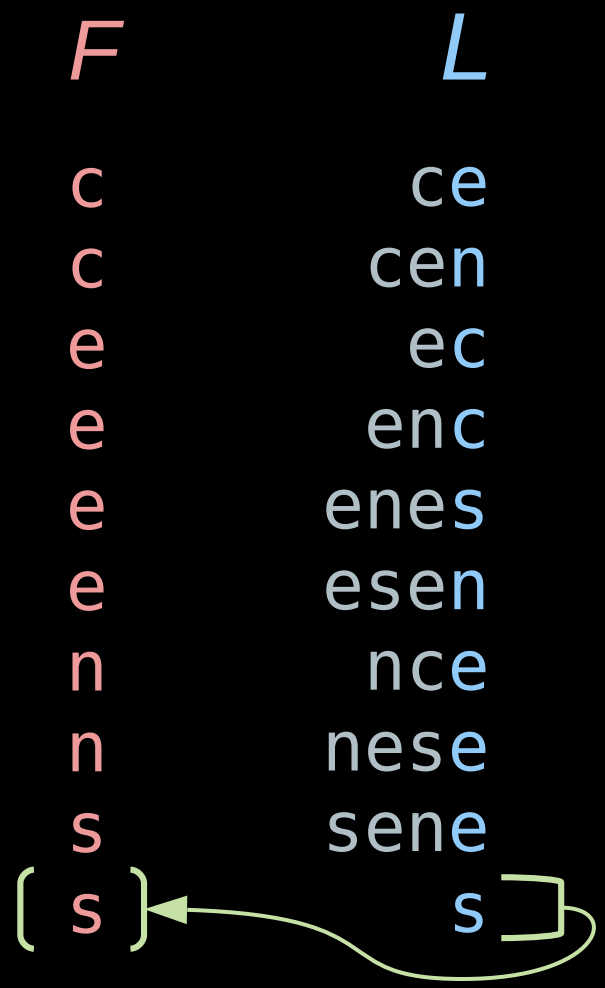
backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

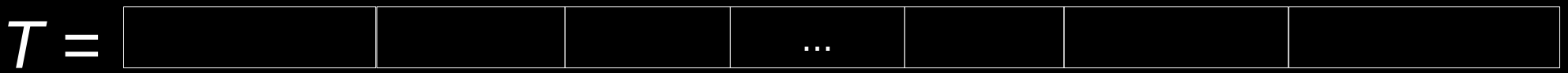
backward search 'ss'

s|enes|cen|ce



- cen is Lyndon word
- ss is **not**

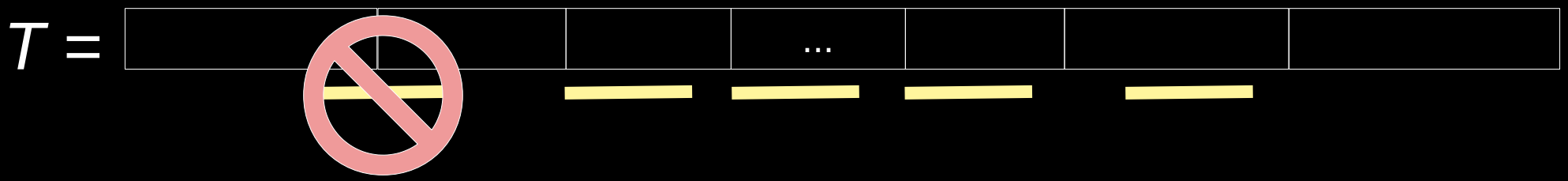
pattern is Lyndon word



pattern is Lyndon word

pattern is Lyndon word

cannot cross Lyndon factor border



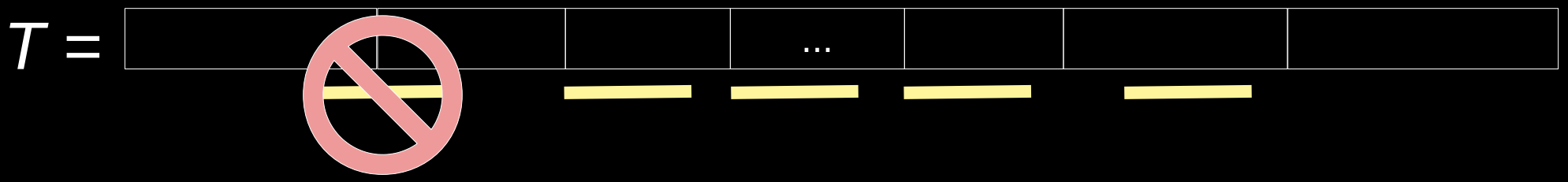
pattern is Lyndon word

cannot cross Lyndon factor border

⇒ occur inside factors

⇒ found within cycles

backward search ≅ FM-index



pattern P is not a Lyndon word

- Lyndon factorization: $P = P_1 \cdots P_m$
- P_y substring of T_x or equal to T_x

algorithm:

- search P_m
- take care when starting with P_{m-1}!

- backward search *P* = se

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

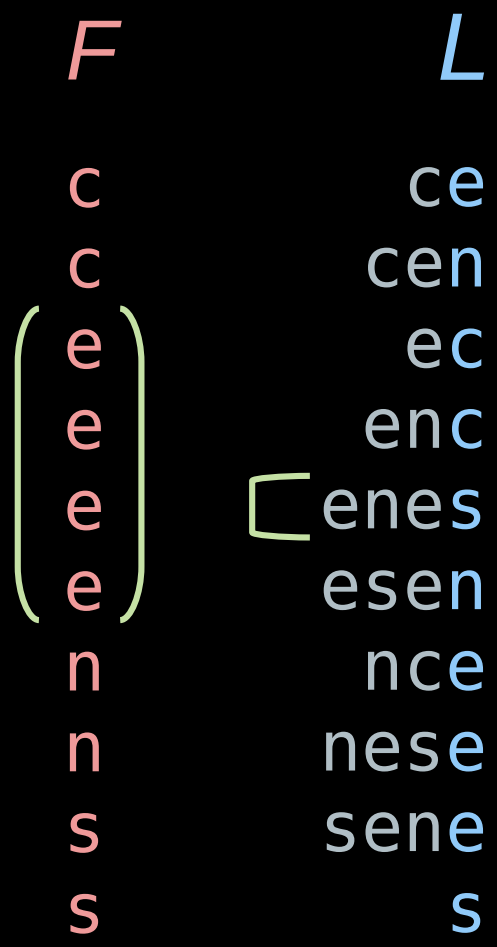
s|enes|cen|ce

- backward search P = se
- P_2 = e

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

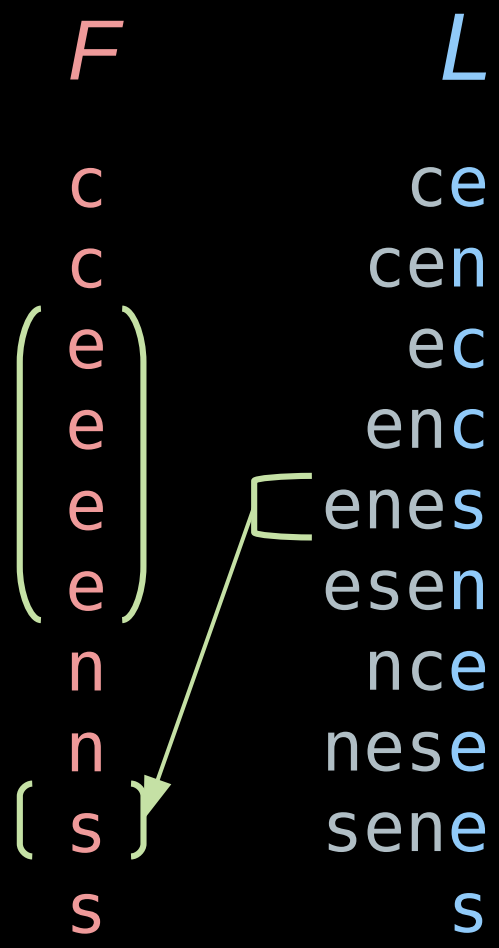
s|enes|cen|ce

- backward search P = se
- P_2 = e



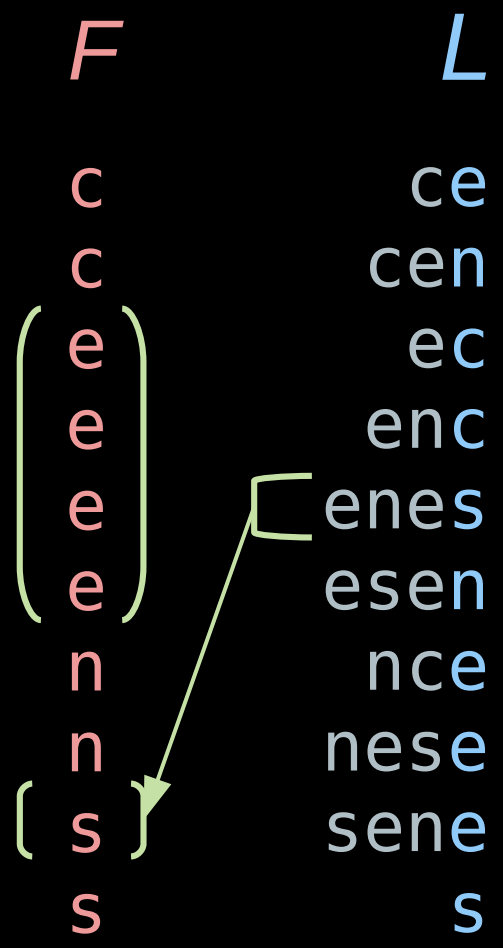
s|enes|cen|ce

- backward search P = se
- P_2 = e
- P_1 = s



s|enes|cen|ce

- backward search P = se
- P_2 = e
- P_1 = s



s|enes|cen|ce

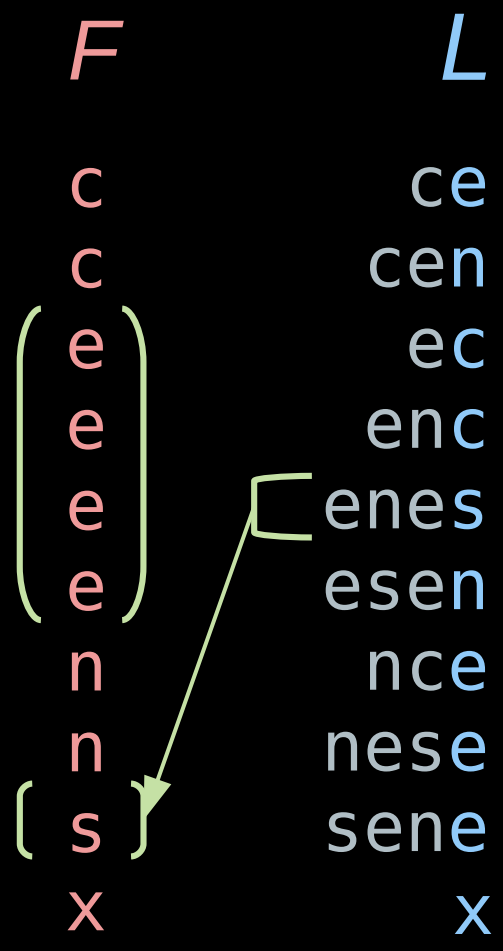
found

s|enes|cen|ce

not counted

false occurrence

- backward search P = se
- P_2 = e
- P_1 = s



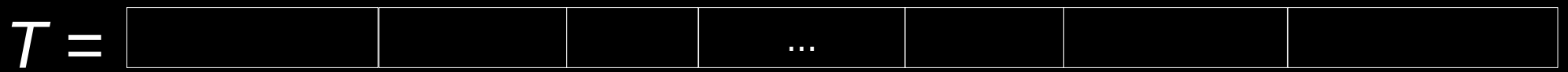
x|enes|cen|ce

found

x|enes|cen|ce

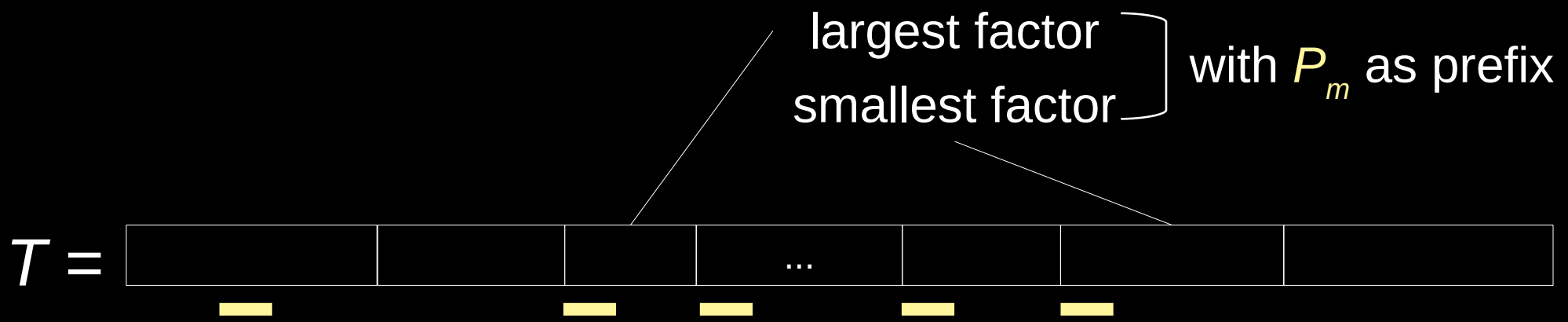
not counted

false occurrence

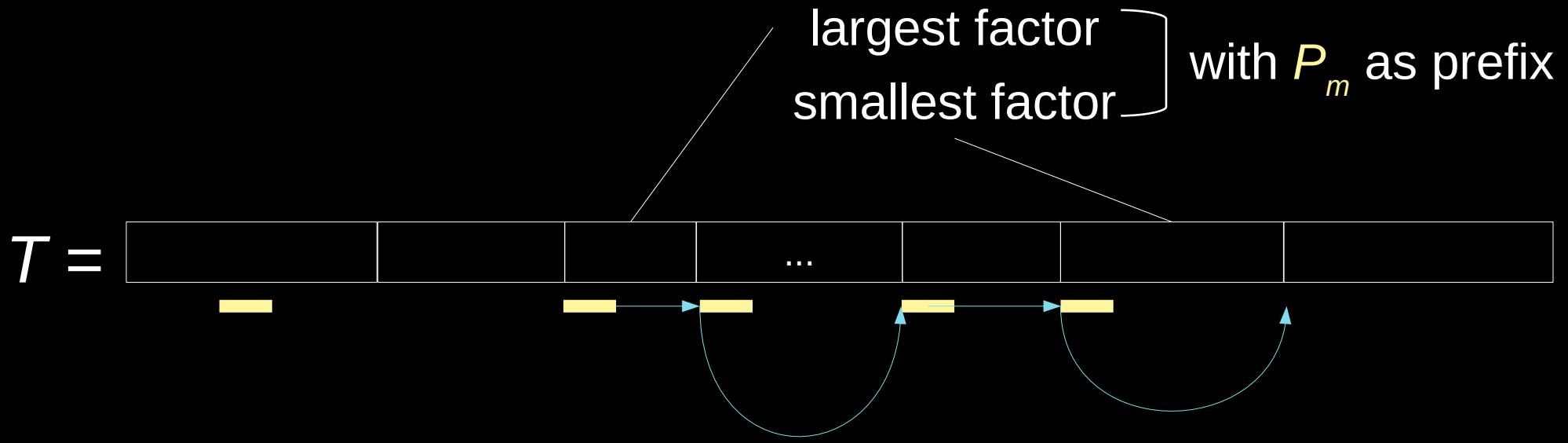

T =
...

- backward search P_m

- backward search P_m



- backward search P_m
- continue search $P_{m-1} P_m$



- backward search P_m
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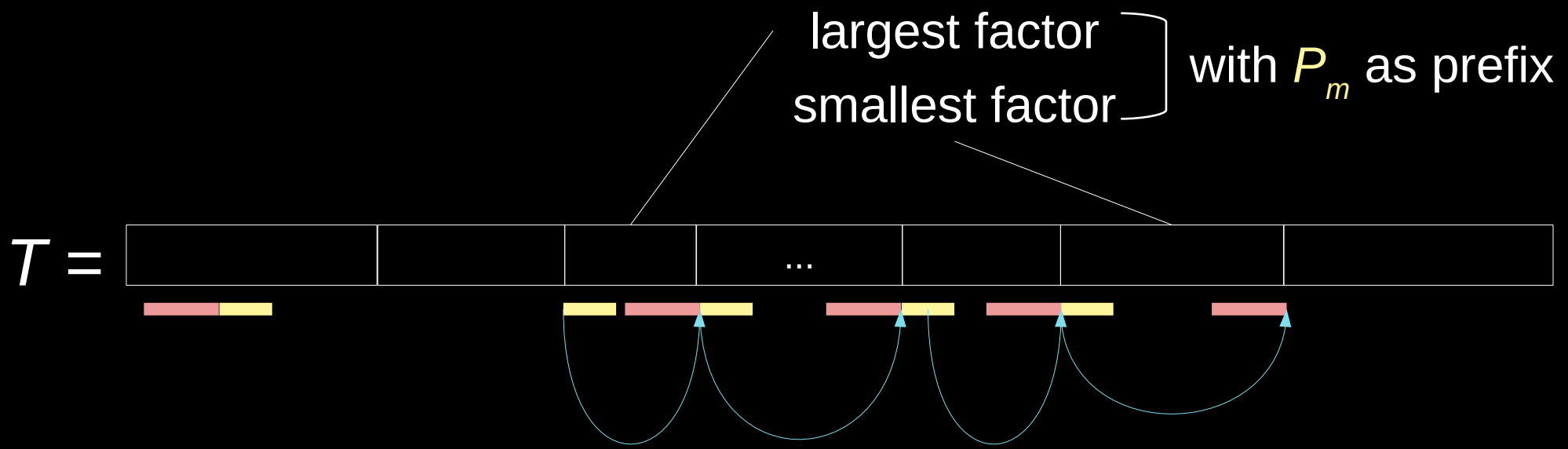

- backward search P_m
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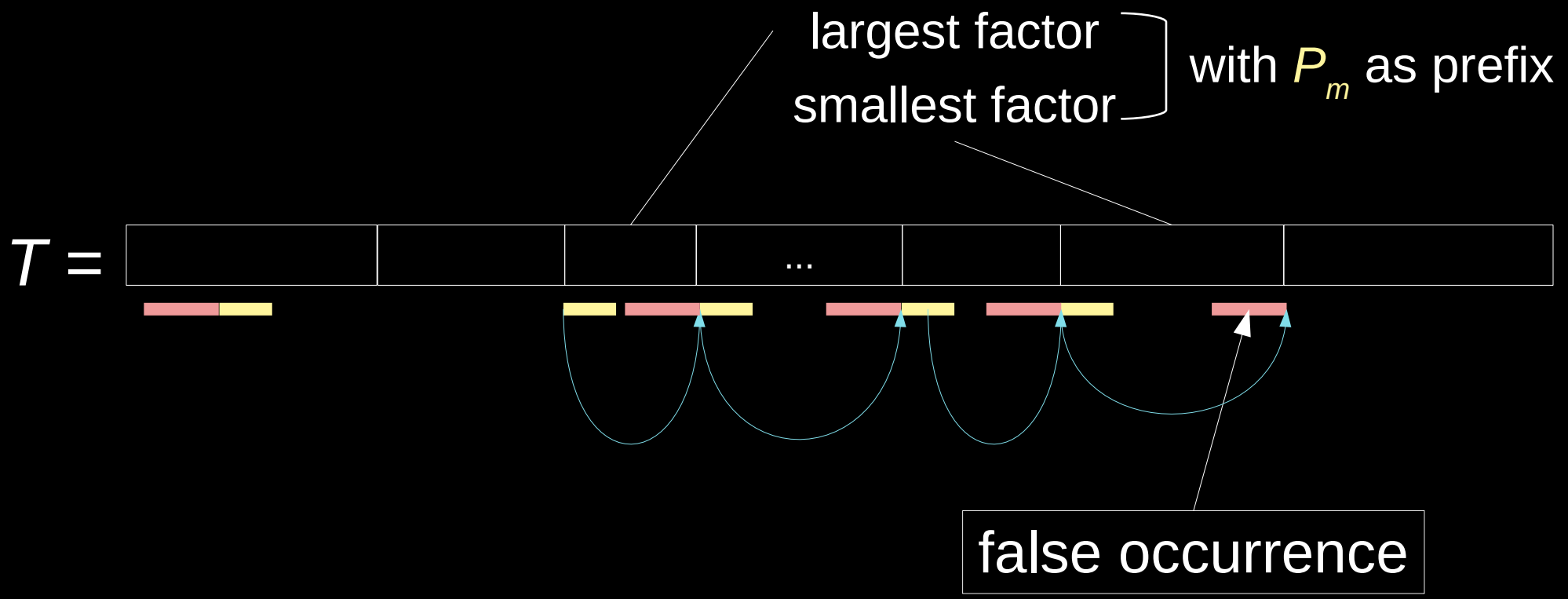

- backward search P_m
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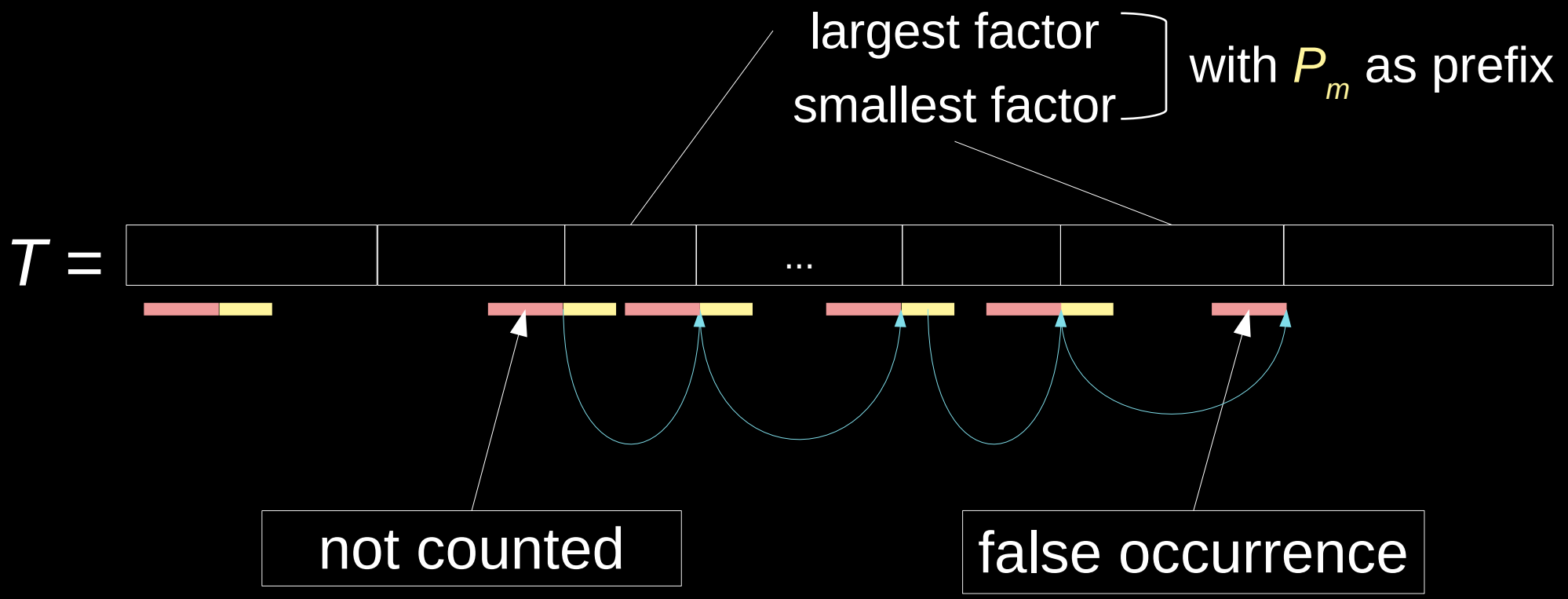
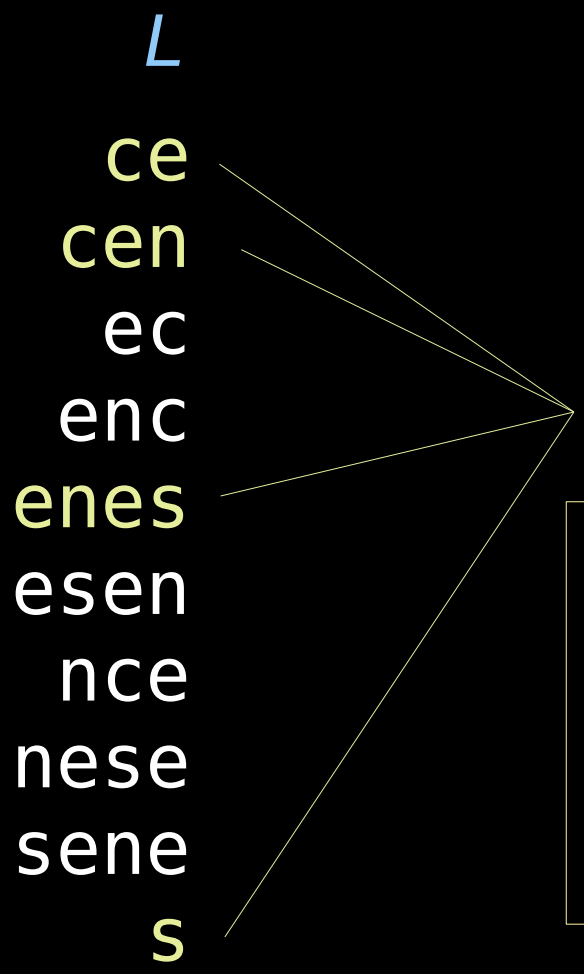


- after finding range of P_m :
  - for border $P_{m-1}P_m$ maintain
    - pointer to not-counted occurrence
    - pointer to false occurrence
- in total backward search on
  - range
  - at most $2m$ individual values
- smallest/largest factor with P_m as prefix = ?

location of factors T_i

s|enes|cen|ce



Lyndon factors T_t, ..., T_1

u, v Lyndon words:
$u <_{\text{lex}} v \iff u \prec_{\omega} v$

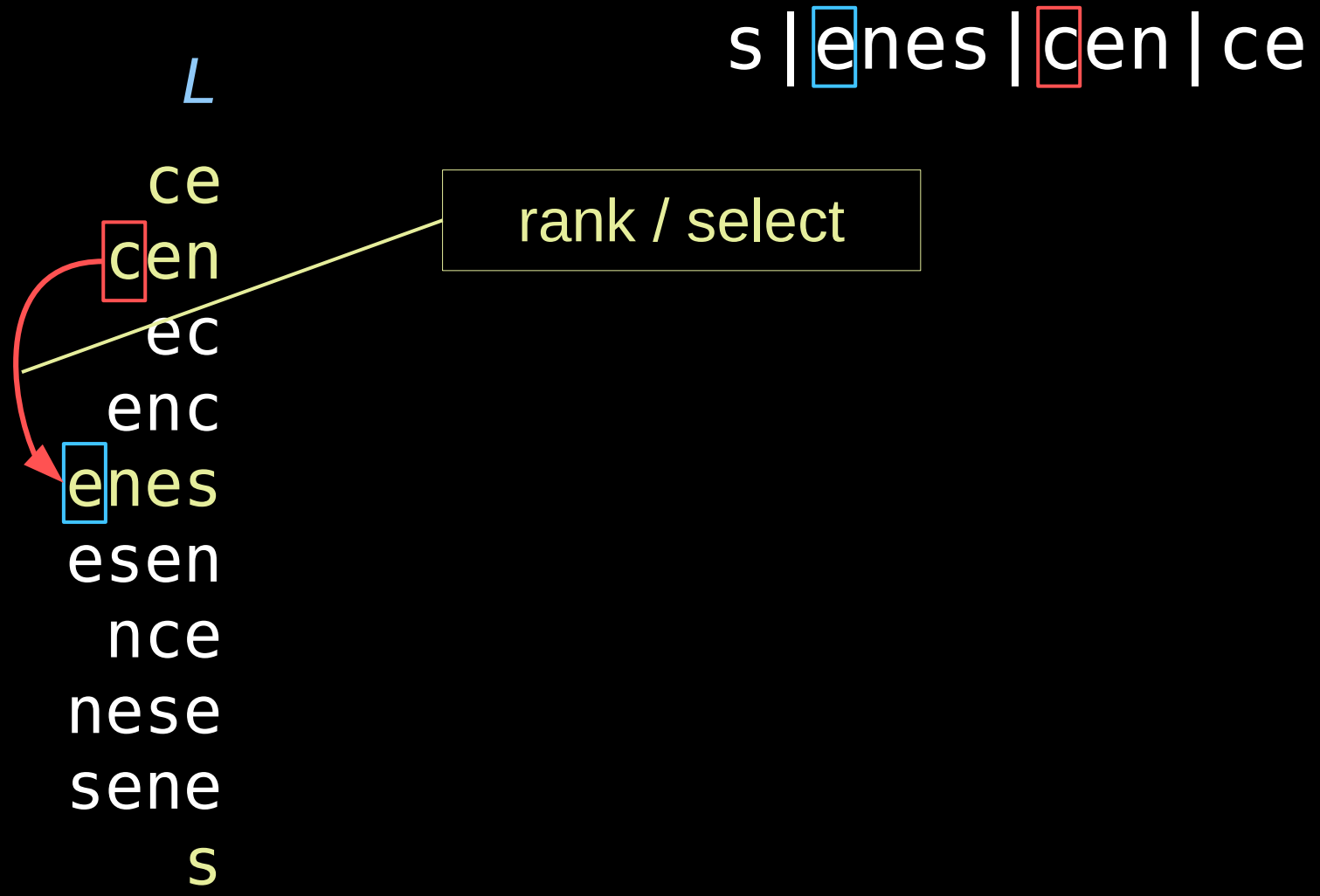
location of factors T_i

s|enes|cen|ce

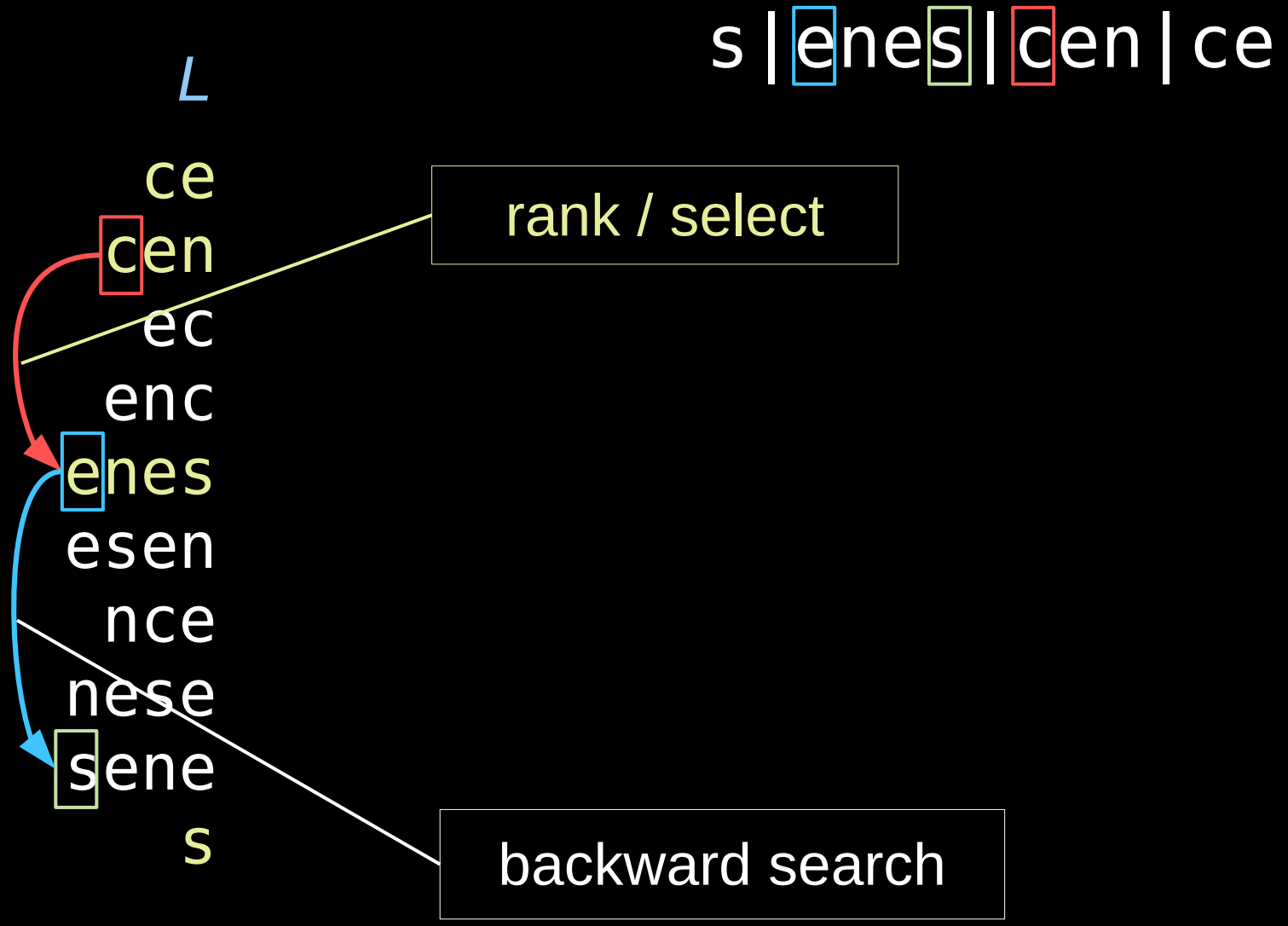
L

ce
cen
ec
enc
enes
esen
nce
nese
sene
s

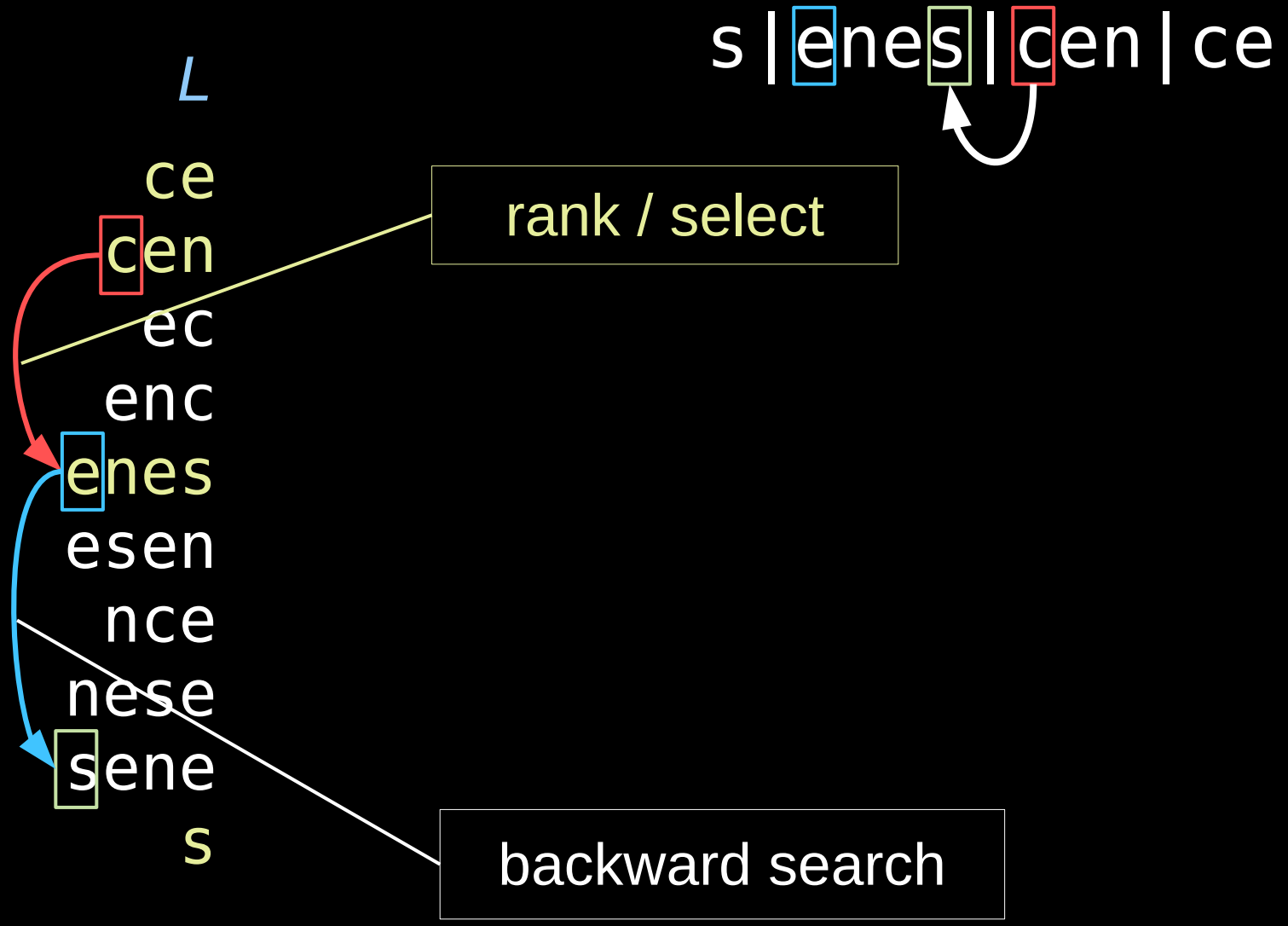
location of factors T_i



location of factors T_i



location of factors T_i



conclusion

- FM index with bijective BWT

  for each pattern character O(lg |P|) additional rank/selects

  ⇒ O(lg |*P*|) times slower than FM index

- uses properties of Lyndon factorization on
  - text
  - pattern *P*

conclusion

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Thank you for your attention. Any questions are welcome!