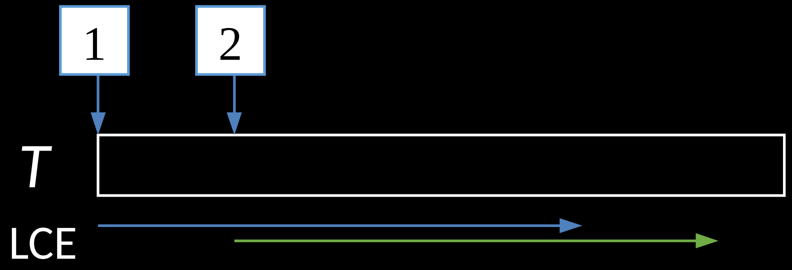
technische universität
dortmund

Deterministic Sparse Suffix Sorting on Rewritable Texts

Johannes Fischer

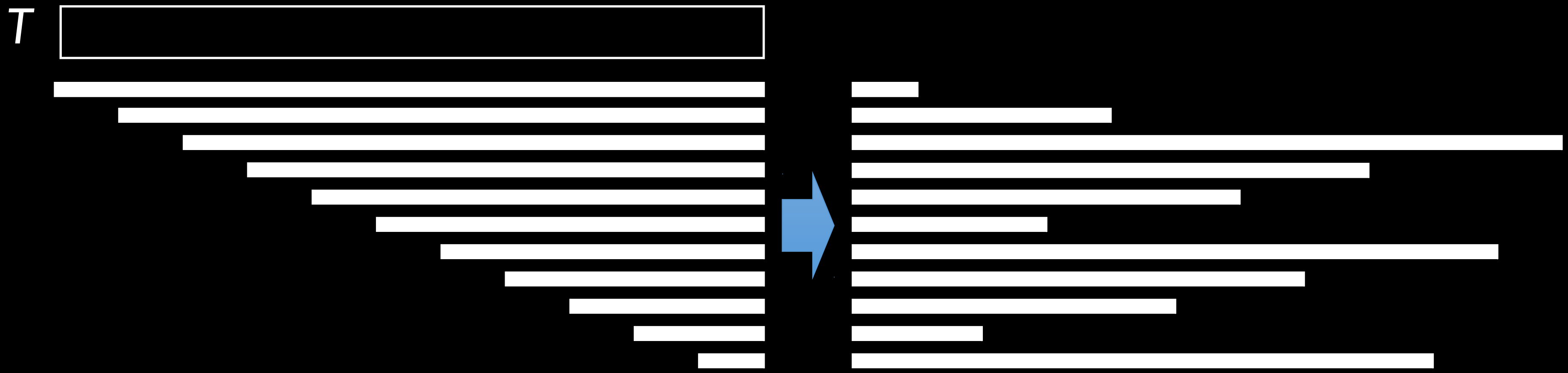
Tomohiro I

Dominik Köppl



suffix sorting problem

- sort all suffixes of string *T* of length *n*
  => suffix array (SA)



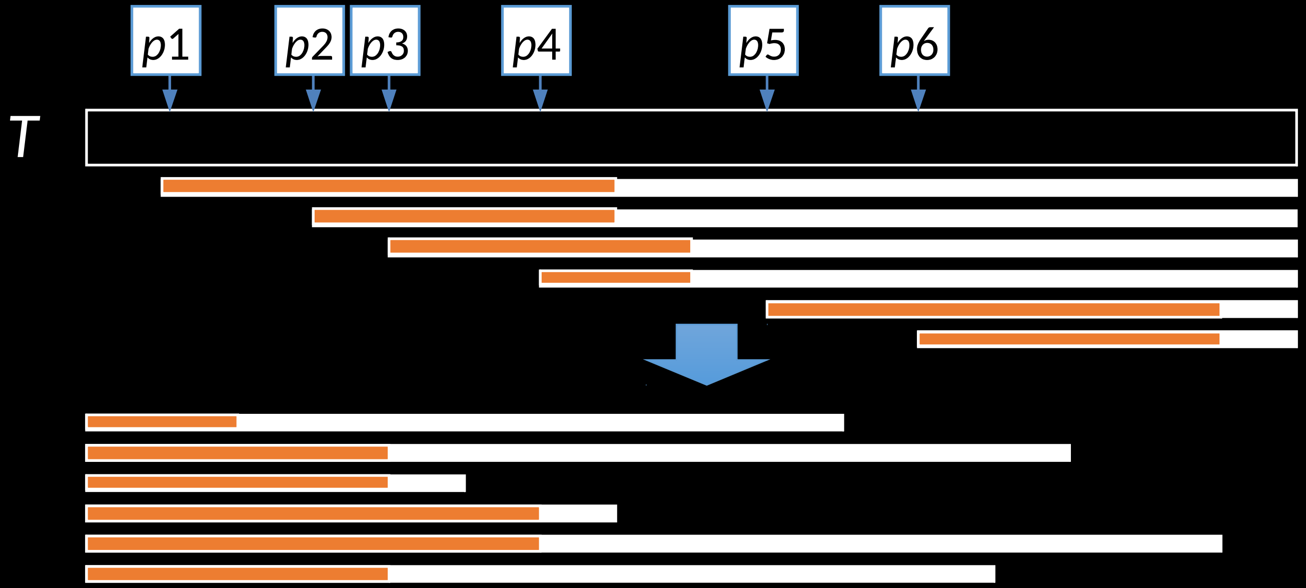
suffix sorting problem

- sort all suffixes of string *T* of length *n*
  => suffix array (SA)
- lengths of the longest common prefix (LCP) between adjacent suffixes

both in *O*(*n*) time and space

sparse suffix sorting problem

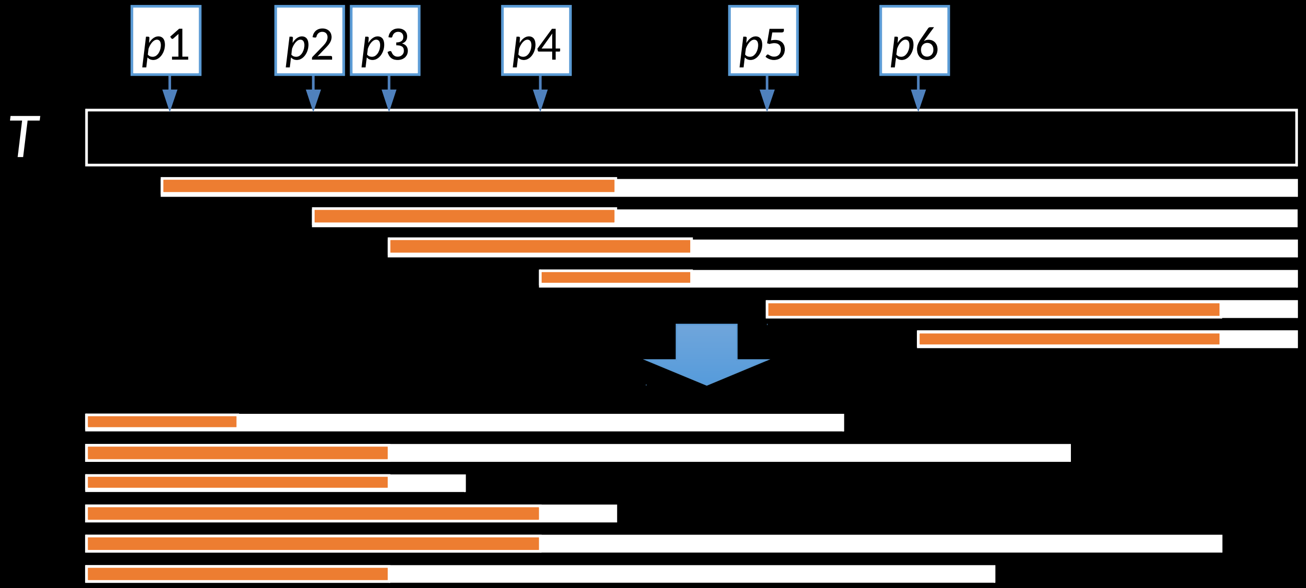
- sort suffixes starting at positions in *P* (+ sparse LCP)
- *P*: a set of *m* (< *n*) text-positions



sparse suffix sorting problem

- sort suffixes starting at positions in P (+ sparse LCP)
- P: a set of m ($< n$) text-positions

  given text in RAM and $m = o(n)$, we want
  - $o(n)$ time
  - $O(m)$ additional space



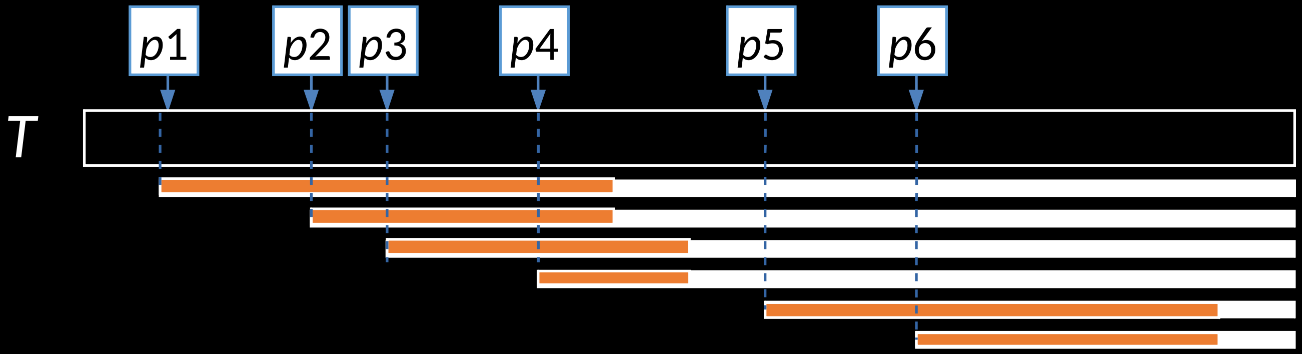
previous work

- $O(n)$-time algos for special P
    - [Kärkkäinen and Ukkonen, ‘96]
    - [Inenaga and Takeda, ‘06]
    - [Ferragina and Fischer, ‘07]
    - [Uemura and Arimura, ‘11]
- arbitrary P (difficult!)
    - [Burkhardt and Kärkkäinen, ‘03]
    - [Bille+, ‘13] randomized
    - [I+, ‘14] improved on Bille+’s work

aim on deterministic algorithms

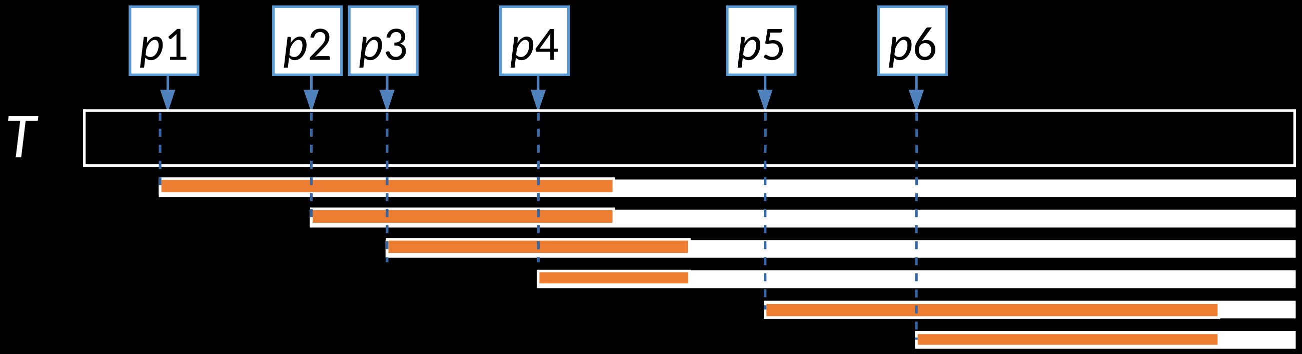
main result

1. text is loaded into RAM



main result

1. text is loaded into RAM
2. text space: re-writeable



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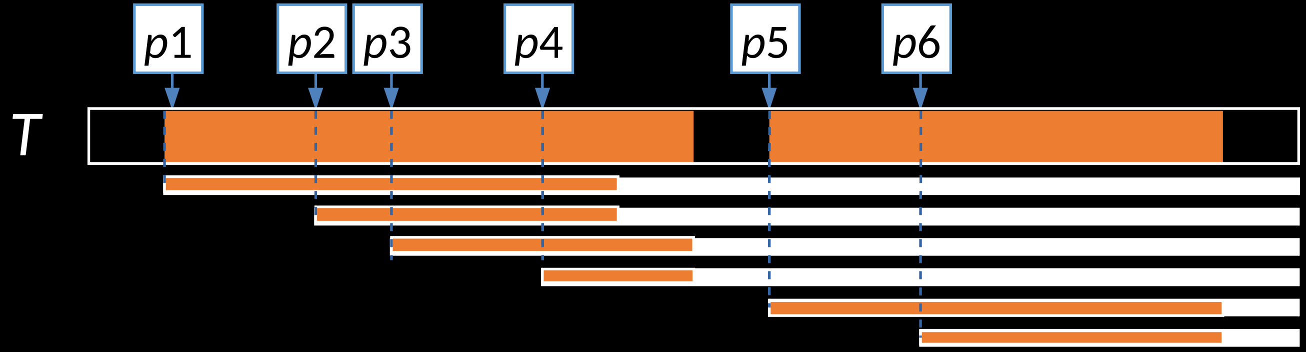
c := # positions that must be compared to sort

$O(m \lg m \lg n \lg^{*} n + c \lg^{0.5} n)$ time
$O(m)$ additional space



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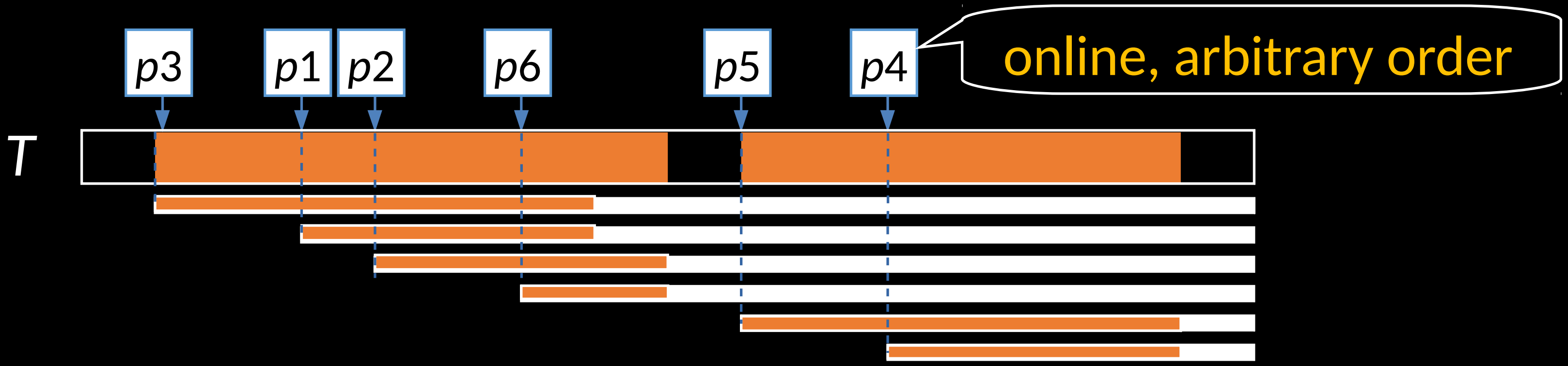

high-level strategy

- use suffix binary search tree (BST) [Irving+,‘03]
- insert every suffix into BST one by one

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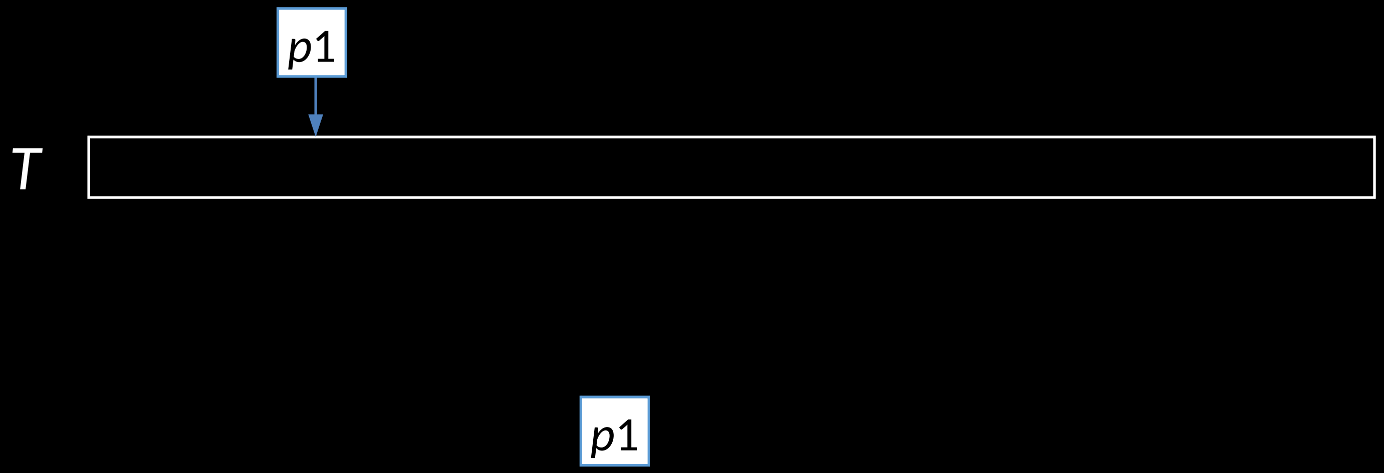

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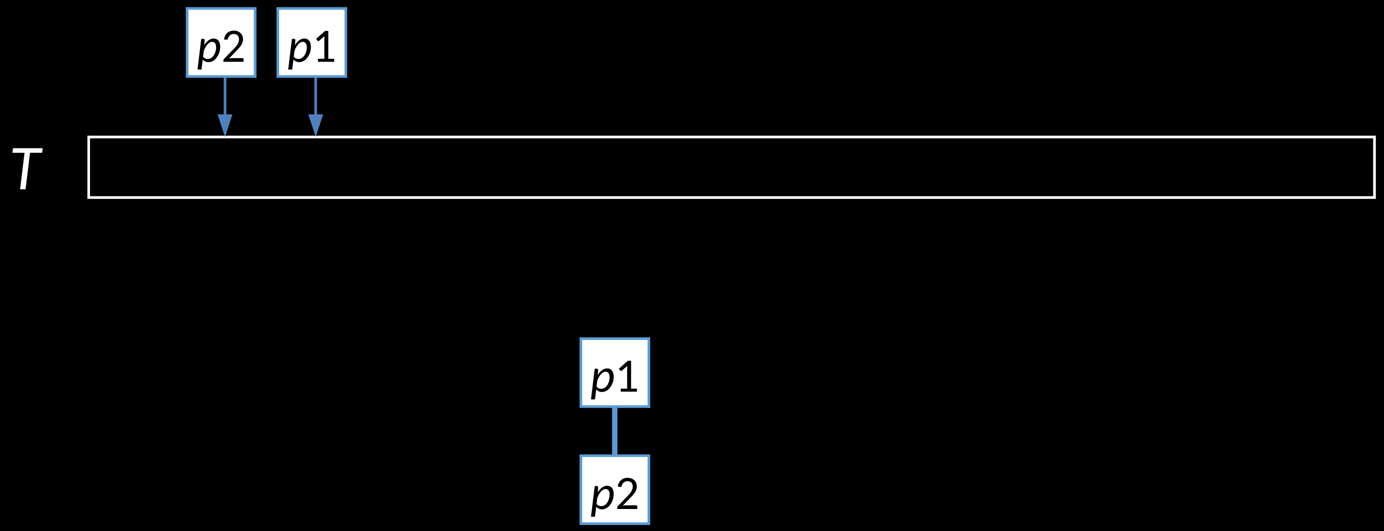

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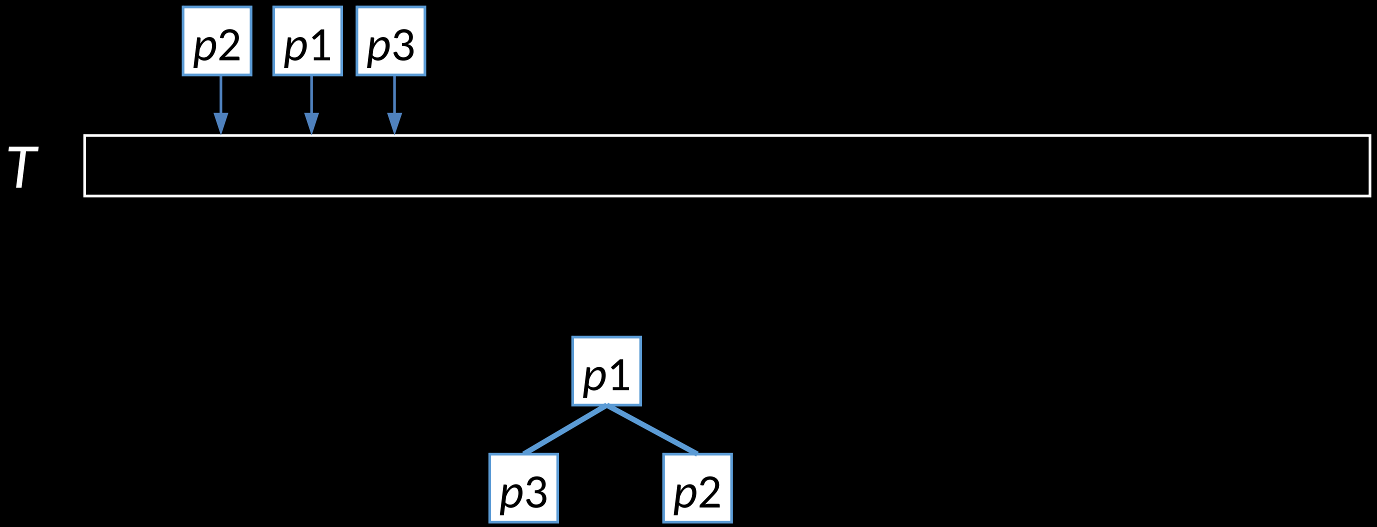

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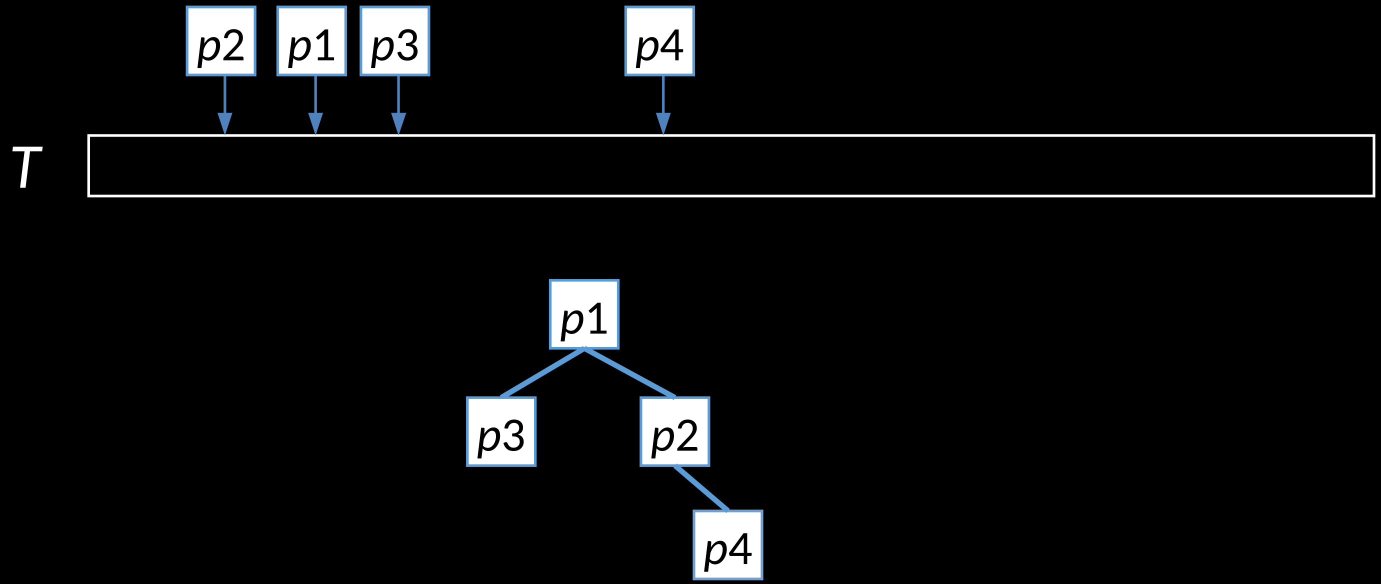
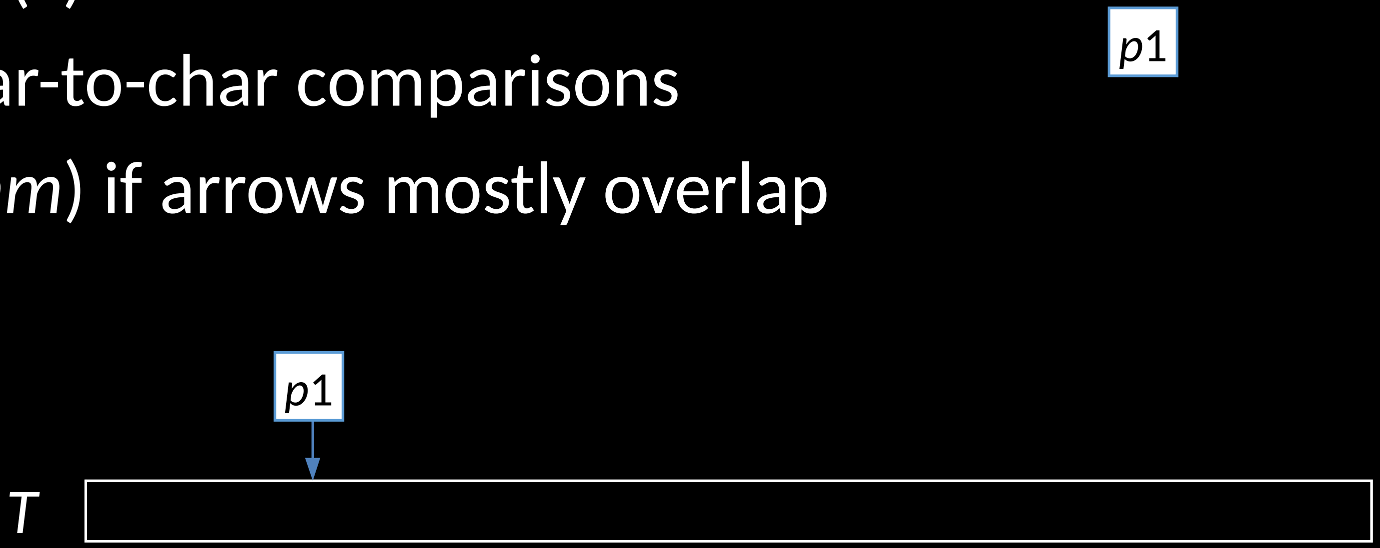


a naïve approach

- takes $O(L)$ time
- L : #char-to-char comparisons
- $L = \Theta(nm)$ if arrows mostly overlap



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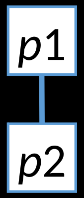

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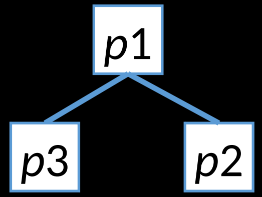

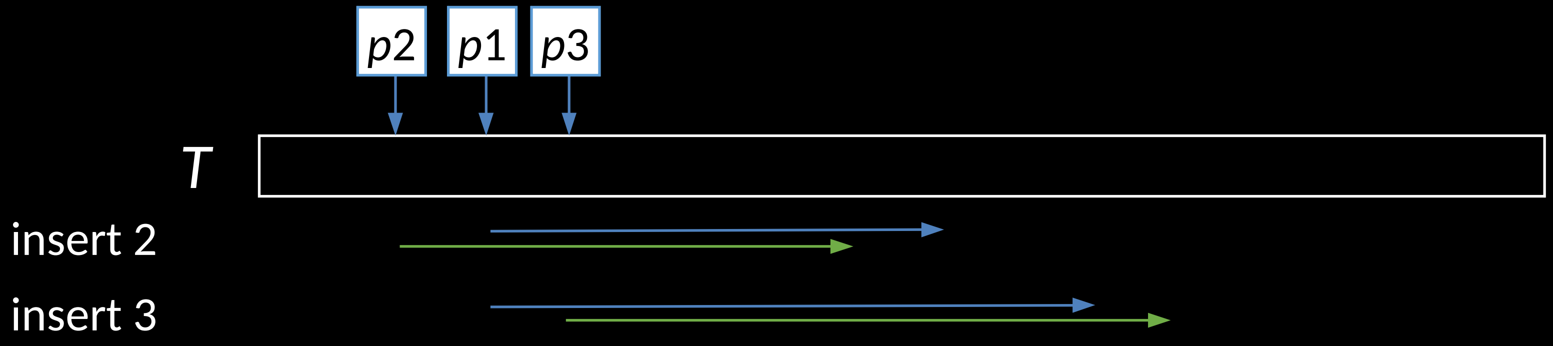

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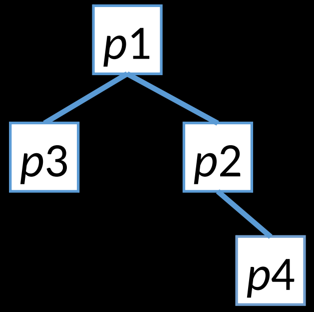
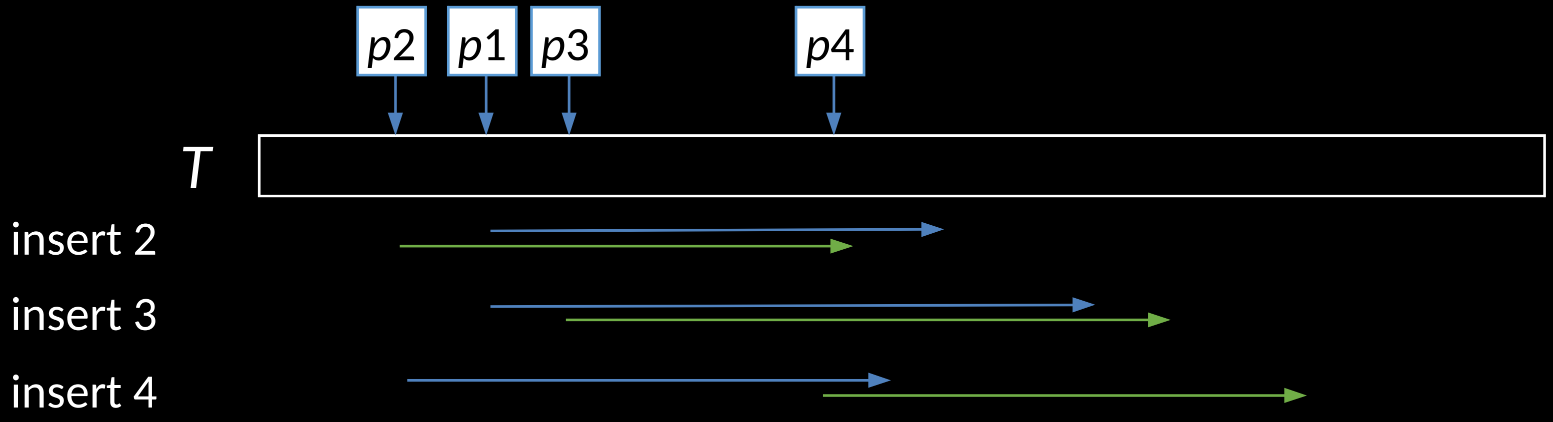
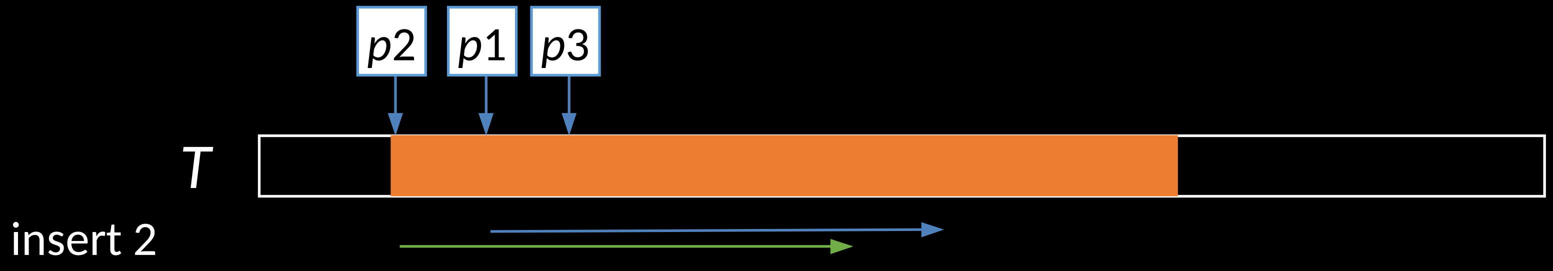




LCE query

- need *c* naive checks
- LCE query:
  longest common prefix of two substrings (suffixes)



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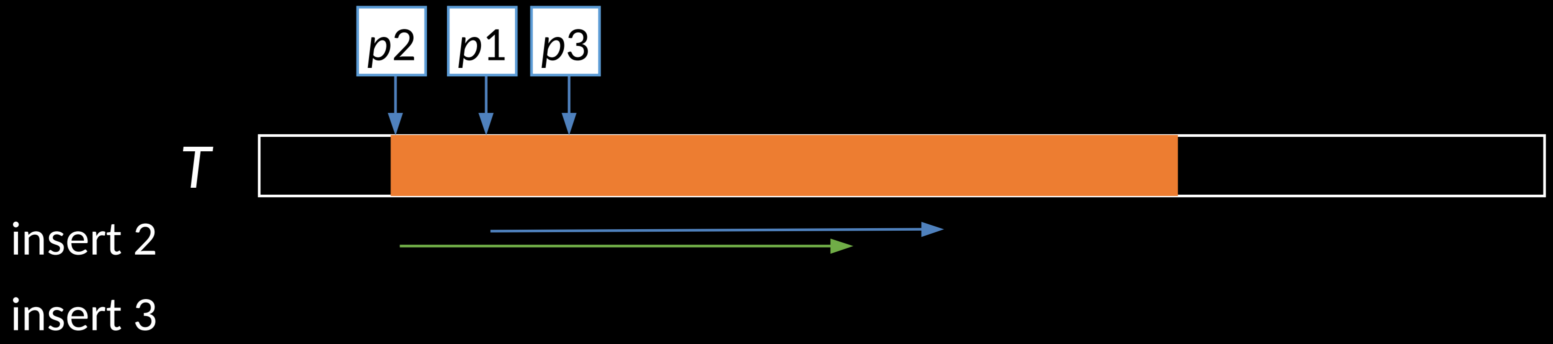

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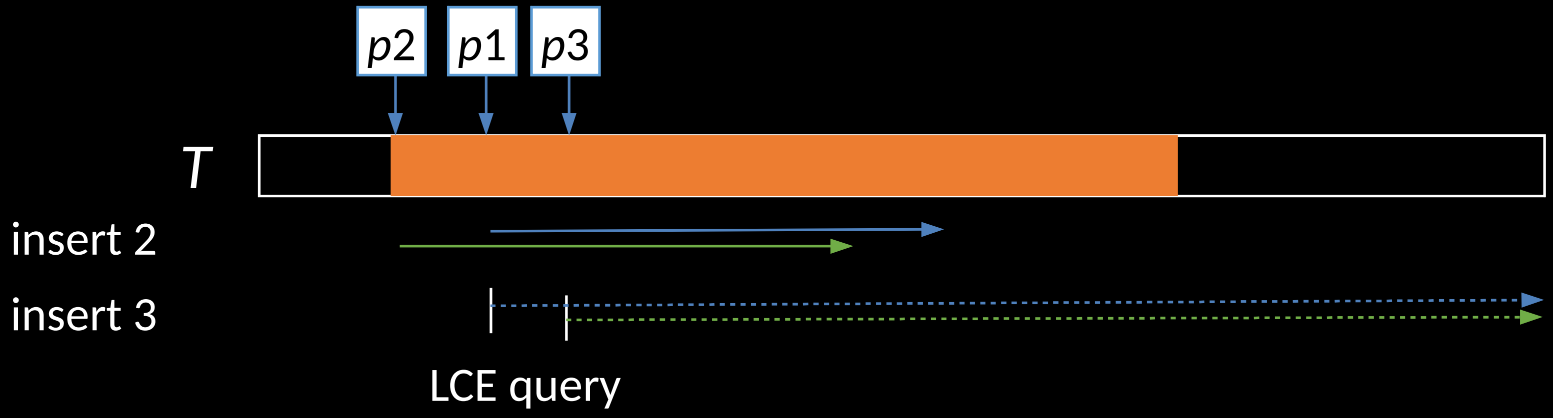

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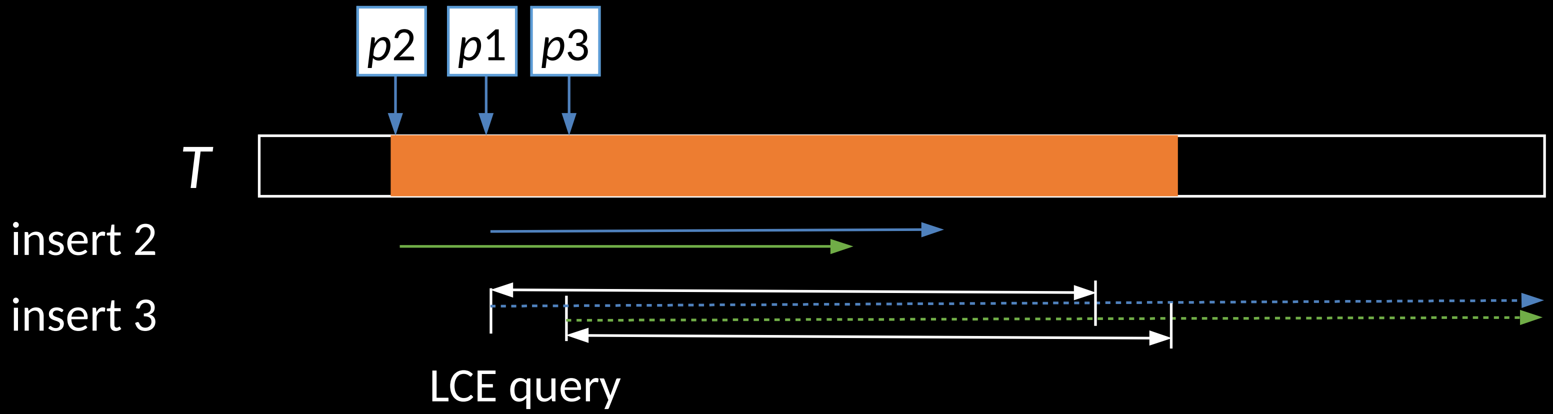

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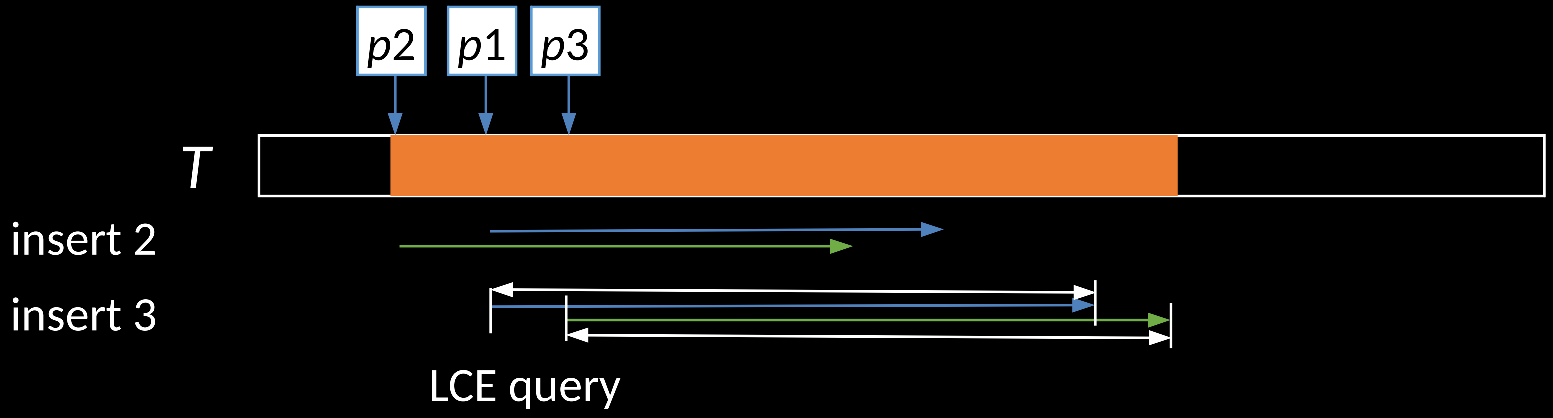

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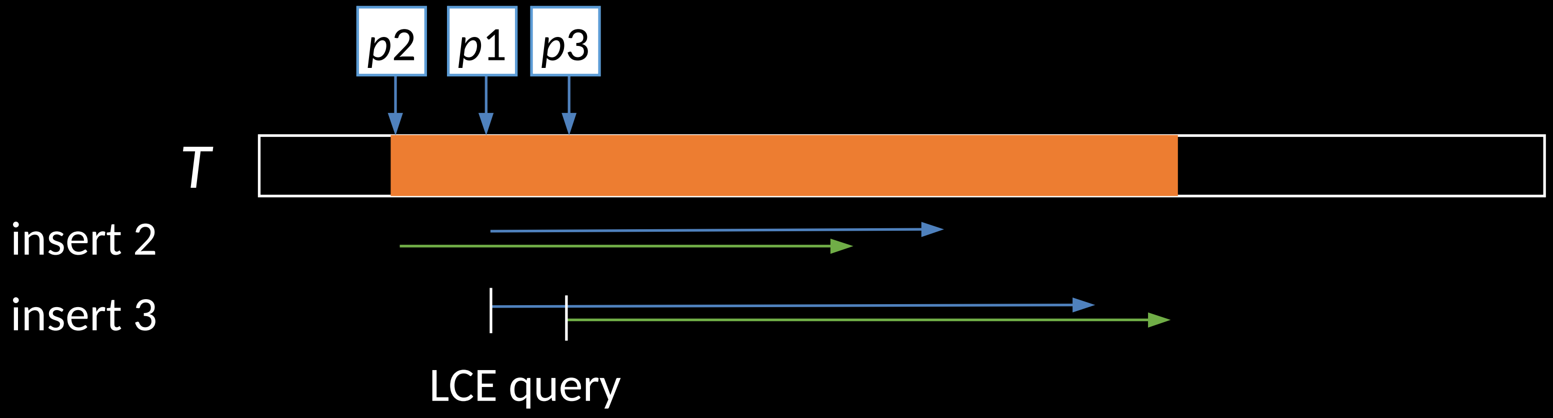
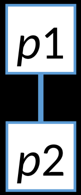
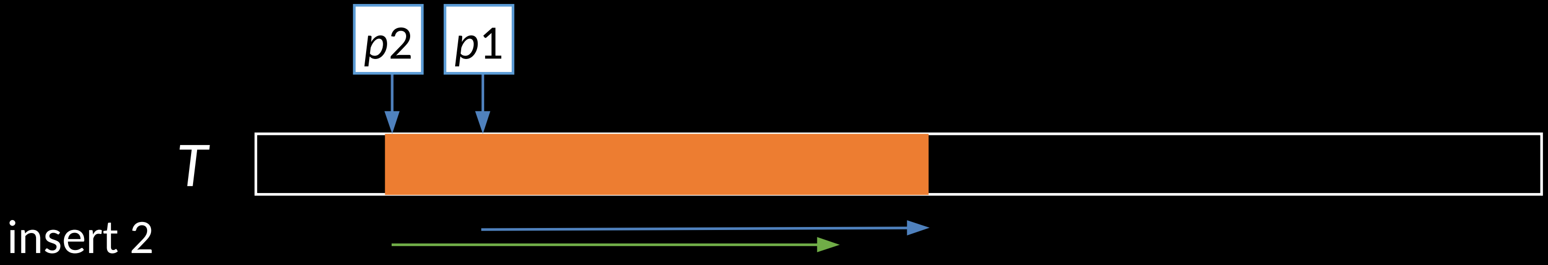


our idea





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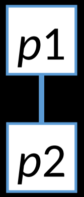
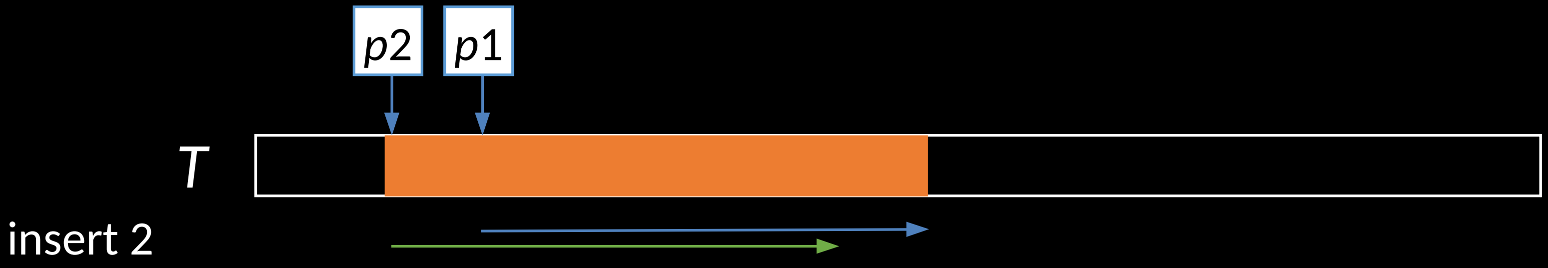


- create an LCE data structure on scanned area



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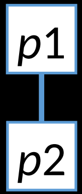

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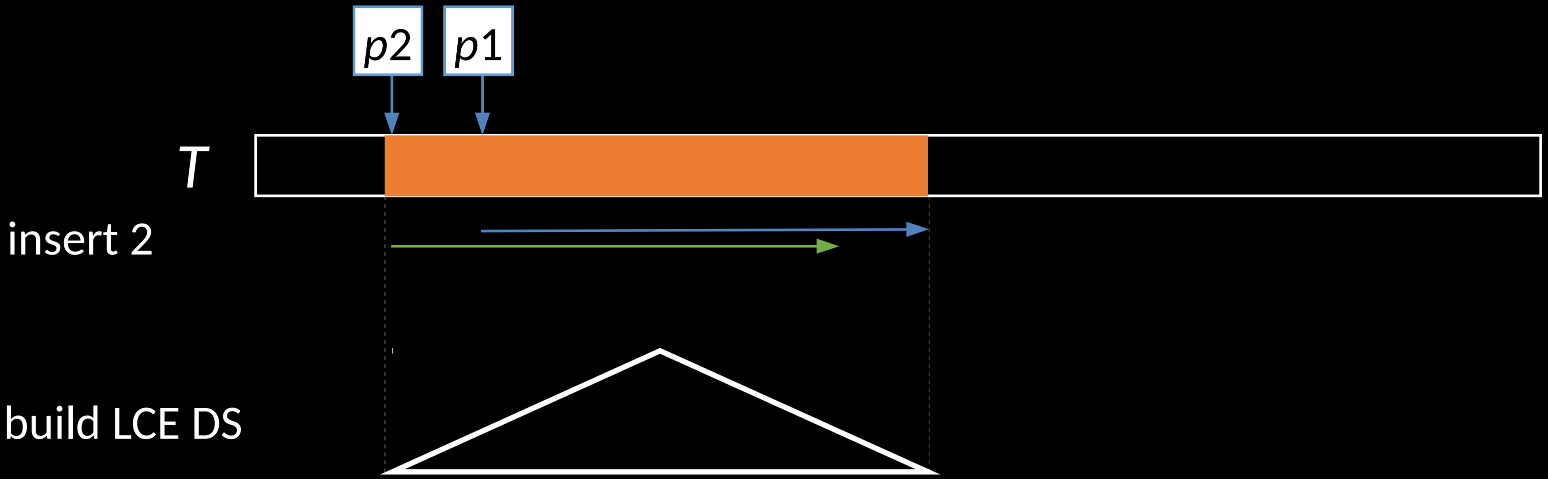

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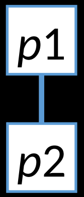

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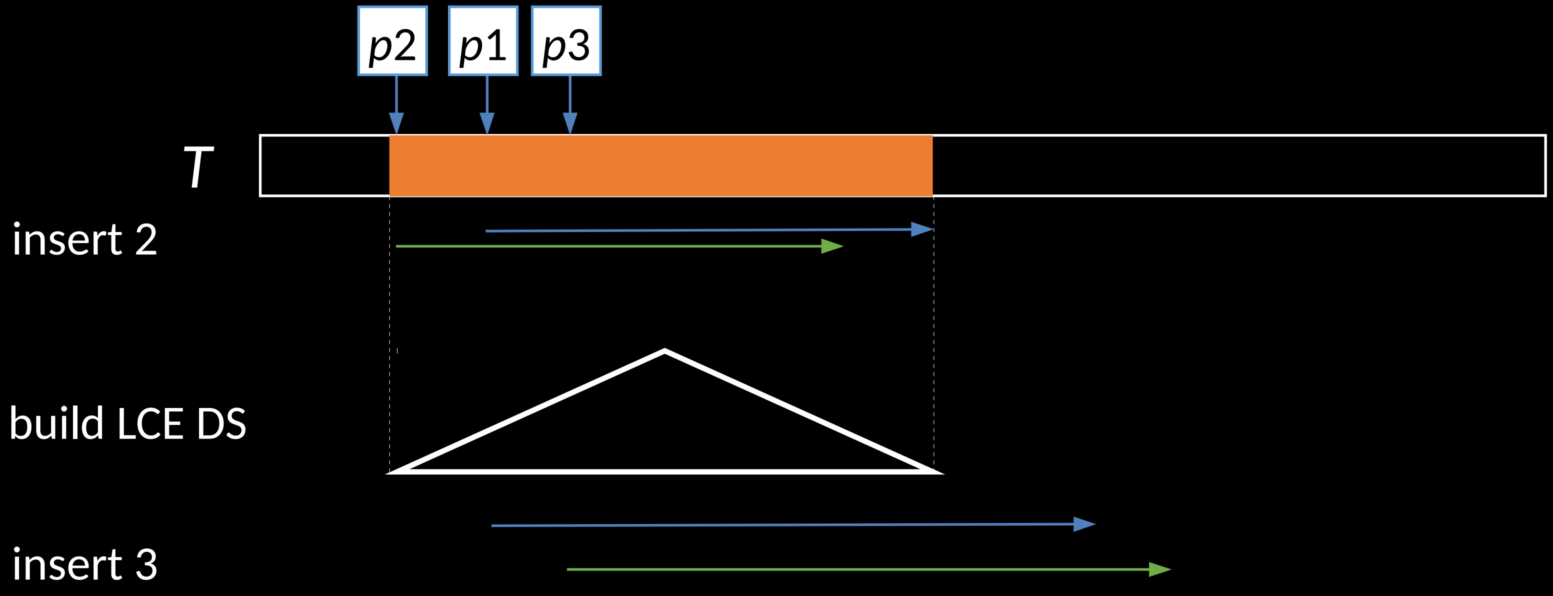

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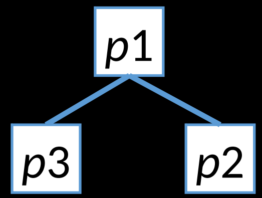
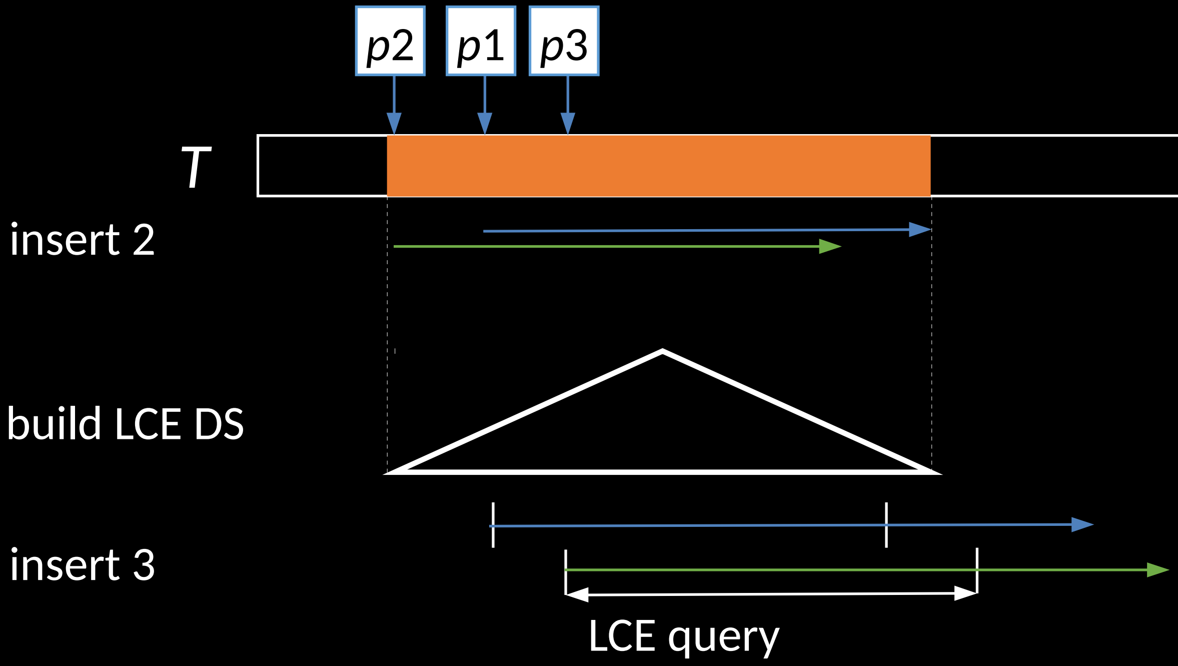


- create an LCE data structure on scanned area
  ⇒ issue LCE queries



ESP = Edit Sensitive Parsing [Cormode+, '07]

LCE with ESP

a b a b a b a b a b c a b a b a b a b a b a b a

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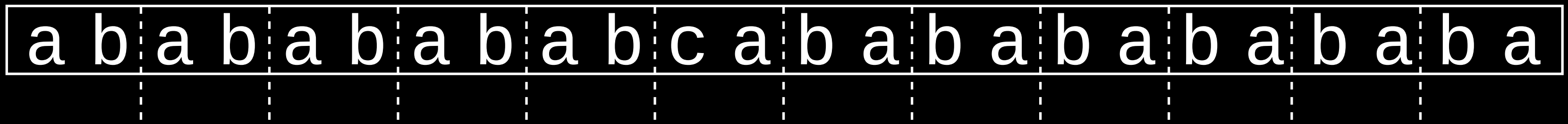
LCE with ESP

- partiton string into blocks of size 2 or 3



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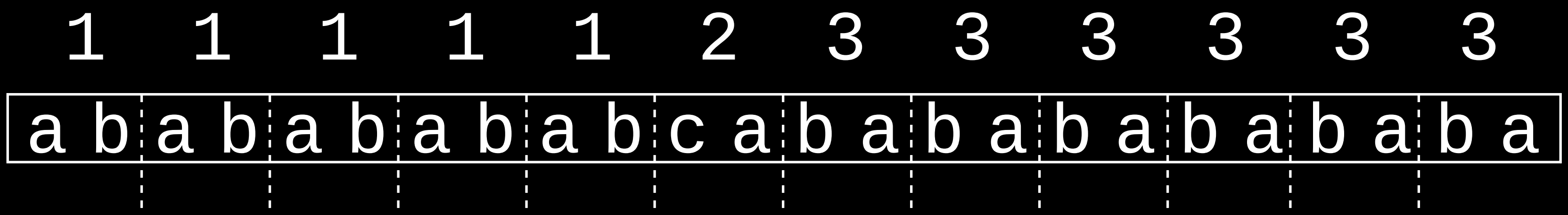
- partiton string into blocks of size 2 or 3
- give each block an ID



1	1	1	1	1	2	3	3	3	3	3	3
a b	a b	a b	a b	a b	c a	b a	b a	b a	b a	b a	b a

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LCE with ESP

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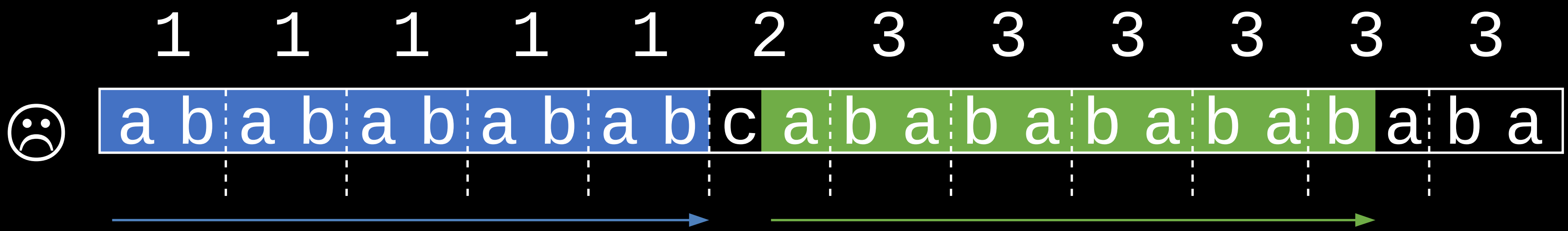
- blocking should be locally consistent



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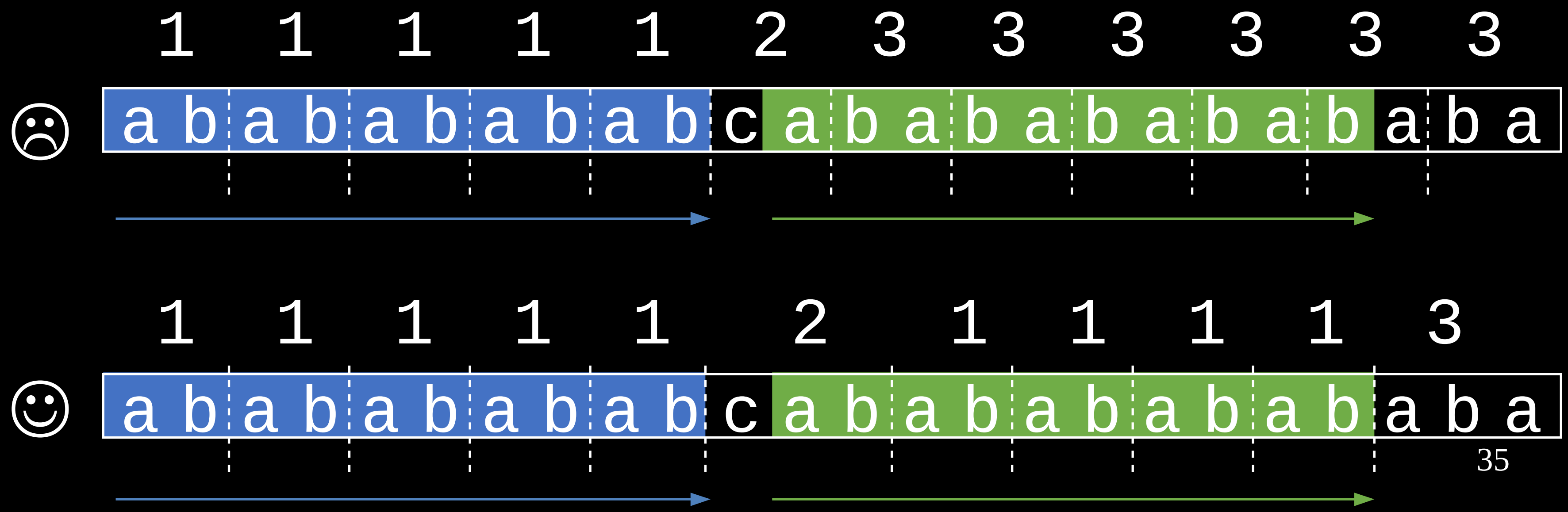

ESP tree

- each block gets an ID (here: number)
- same content ⇔ same ID

ababbabababababbababaab

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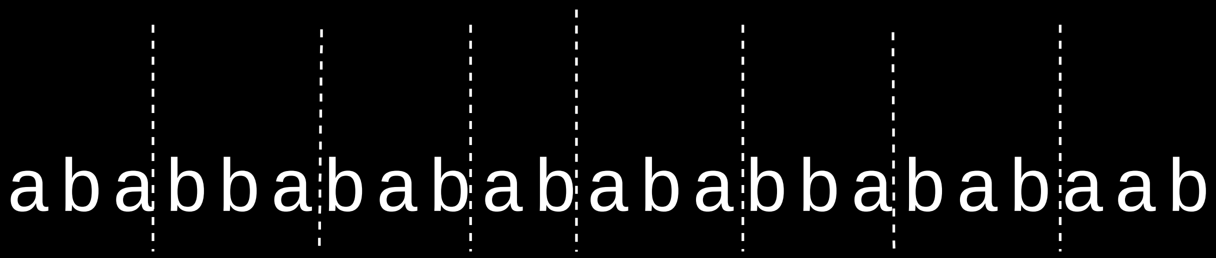

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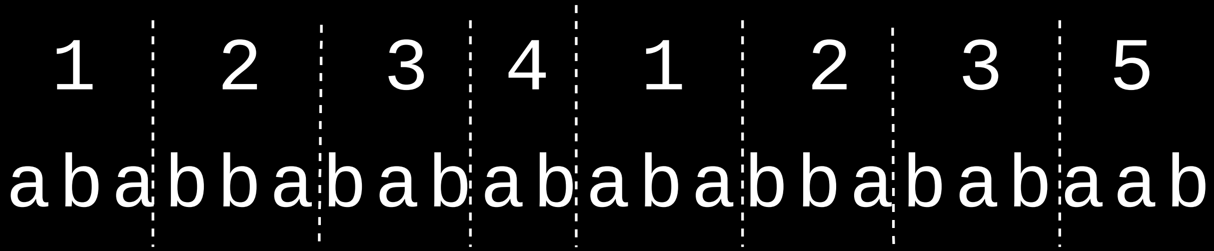

1 2 3 4 1 2 3 5

a b a b b a b a b a b a b a b b a b a b a a b

binary search tree →

ID		string
1	→	a b a
2	→	b b a
3	→	b a b
4	→	a b
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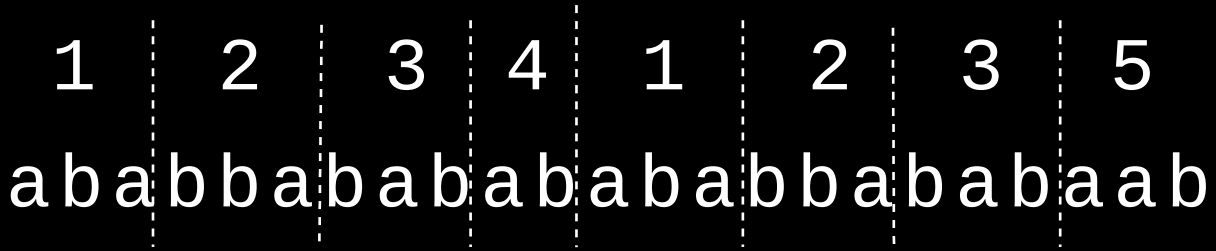

ESP tree

- each block gets an ID (here: number)
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- recursion spans up ESP tree

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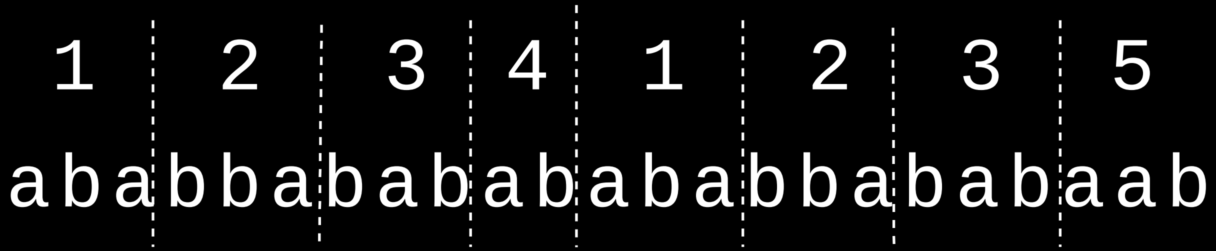

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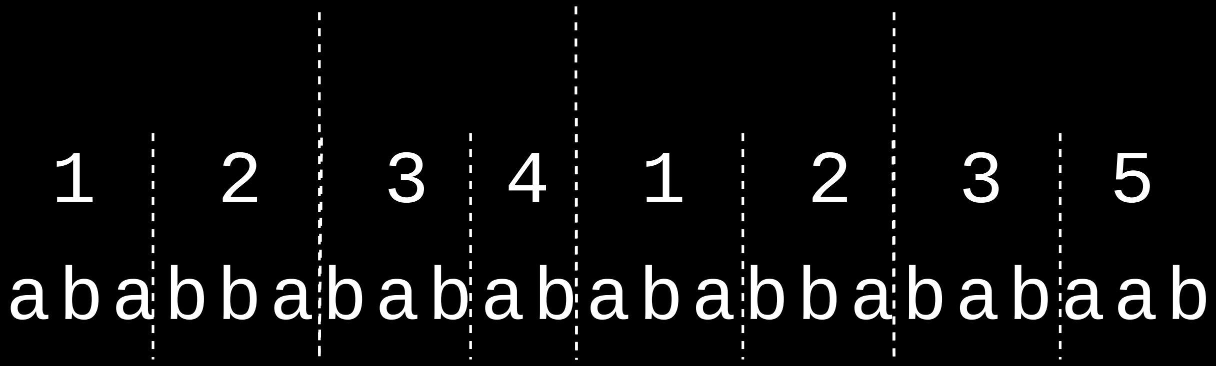

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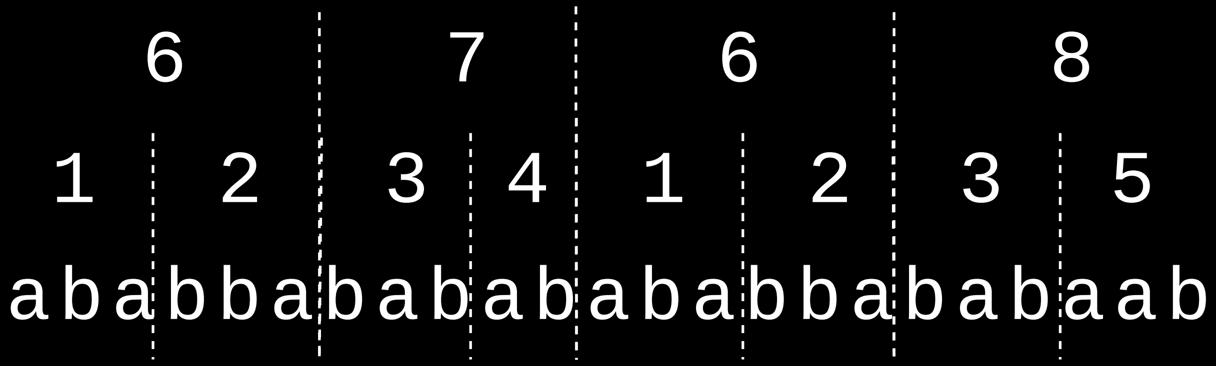

6 7 6 8

1 2 3 4 1 2 3 5

a b a b b a b a b a b a b a b b a b a b a a b

ESP tree

binary search tree →

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1	→	a b a
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5	→	a a b
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7	→	3 4
8	→	3 5

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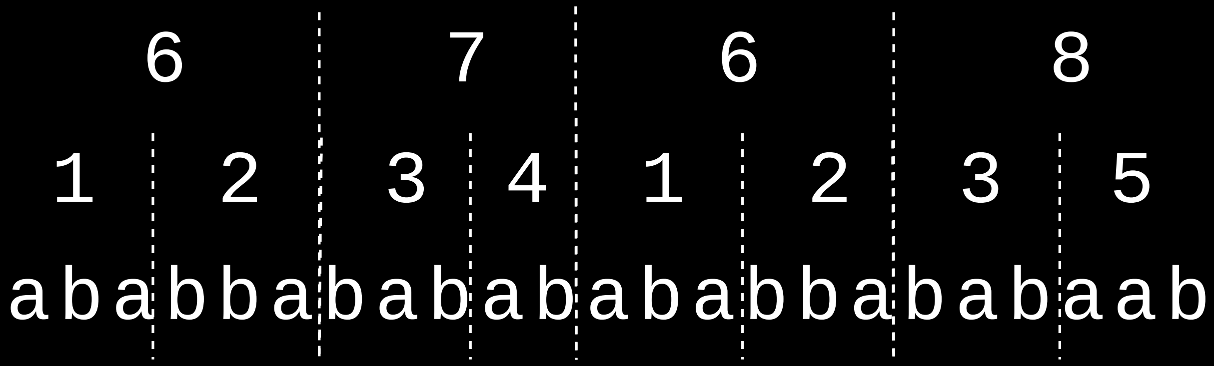

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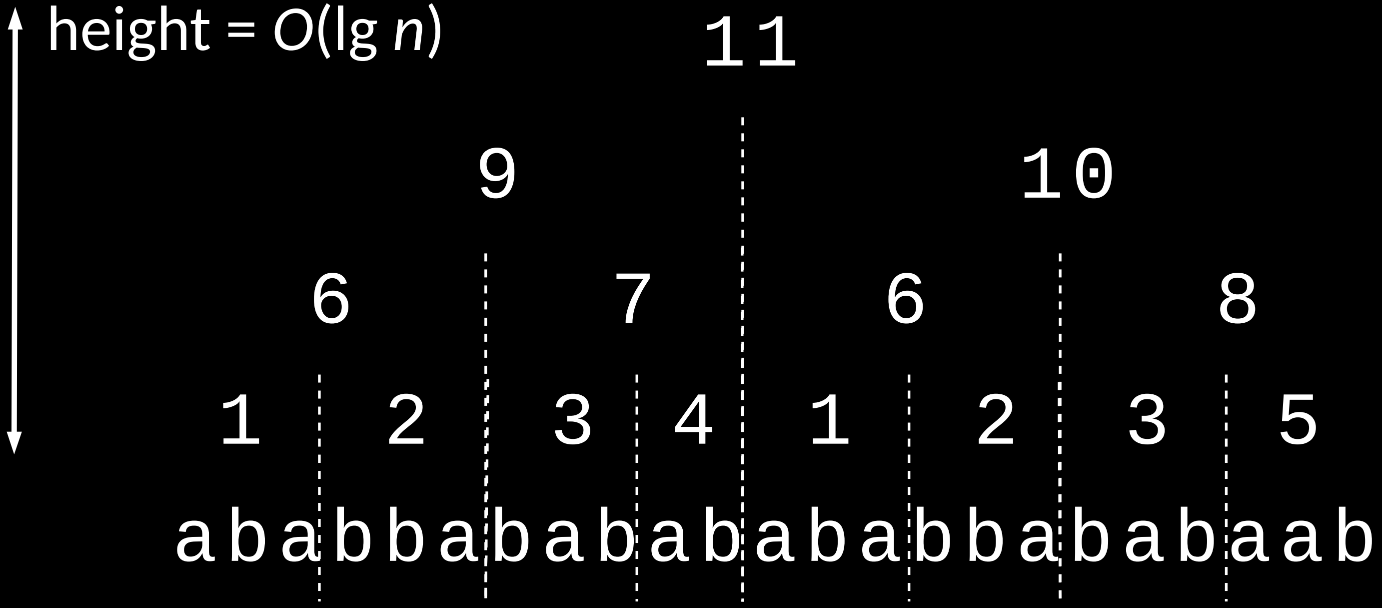
9	→	6 7
10	→	6 8
11	→	9 10



construction time

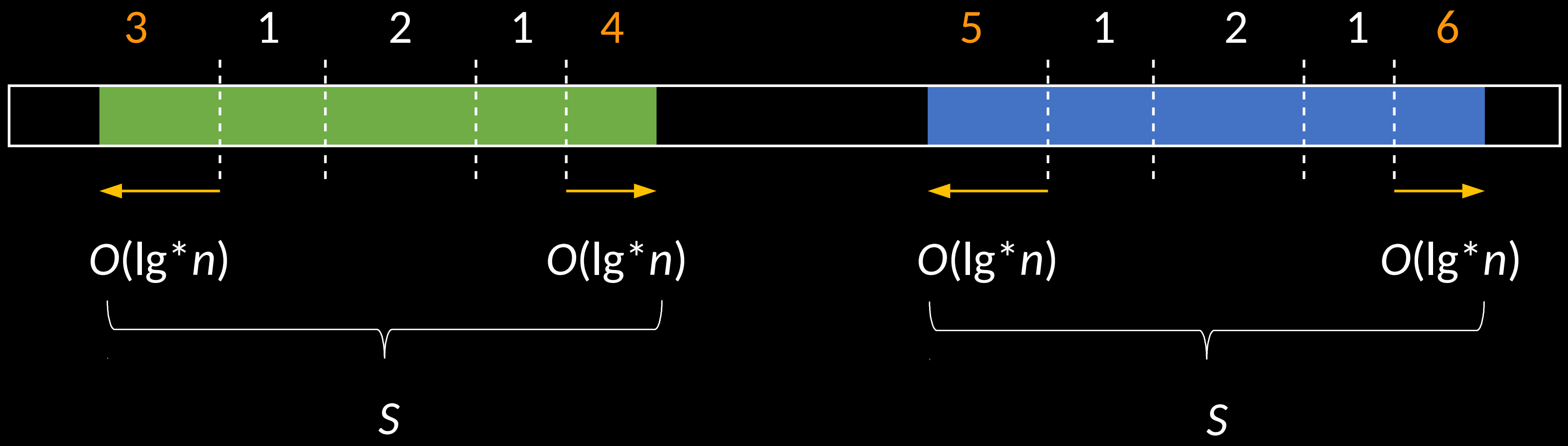
- $O(\lg n)$ time lookup/store IDs in dictionary
- $O(\lg^* n)$ time for blocking [Cormode+, '07]

$\Rightarrow O(|S|(\lg n + \lg^* n)) = O(|S| \lg n)$ time

for ESP tree on substr S

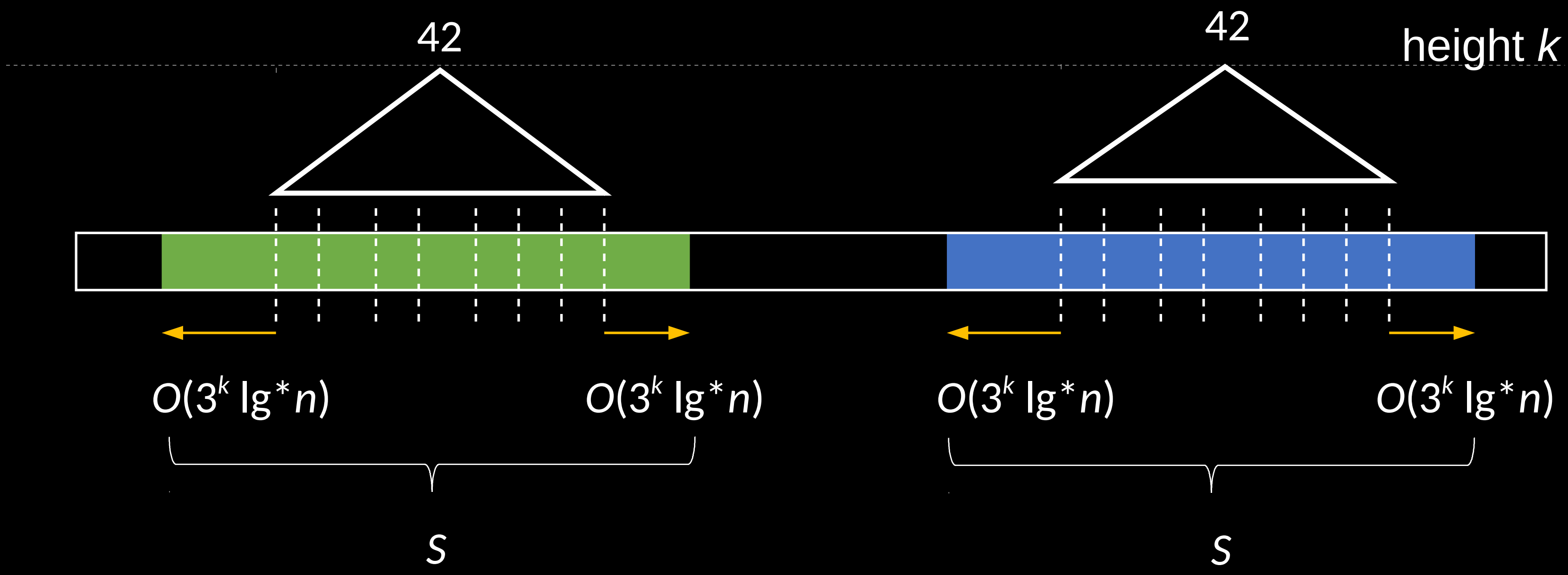
local surrounding

- two equal substrings *S*
- sufficiently enclosed nodes have same ID

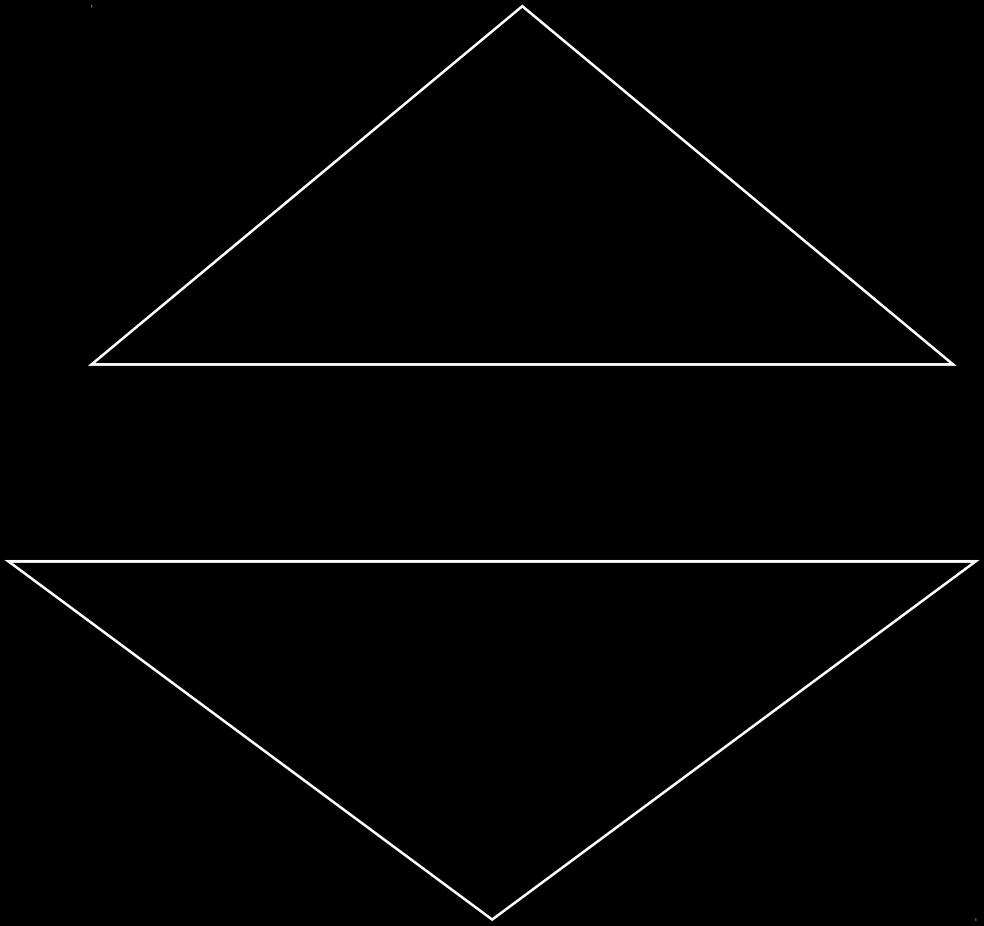


local surrounding

- two equal substrings S
- sufficiently enclosed ESP subtree have same root

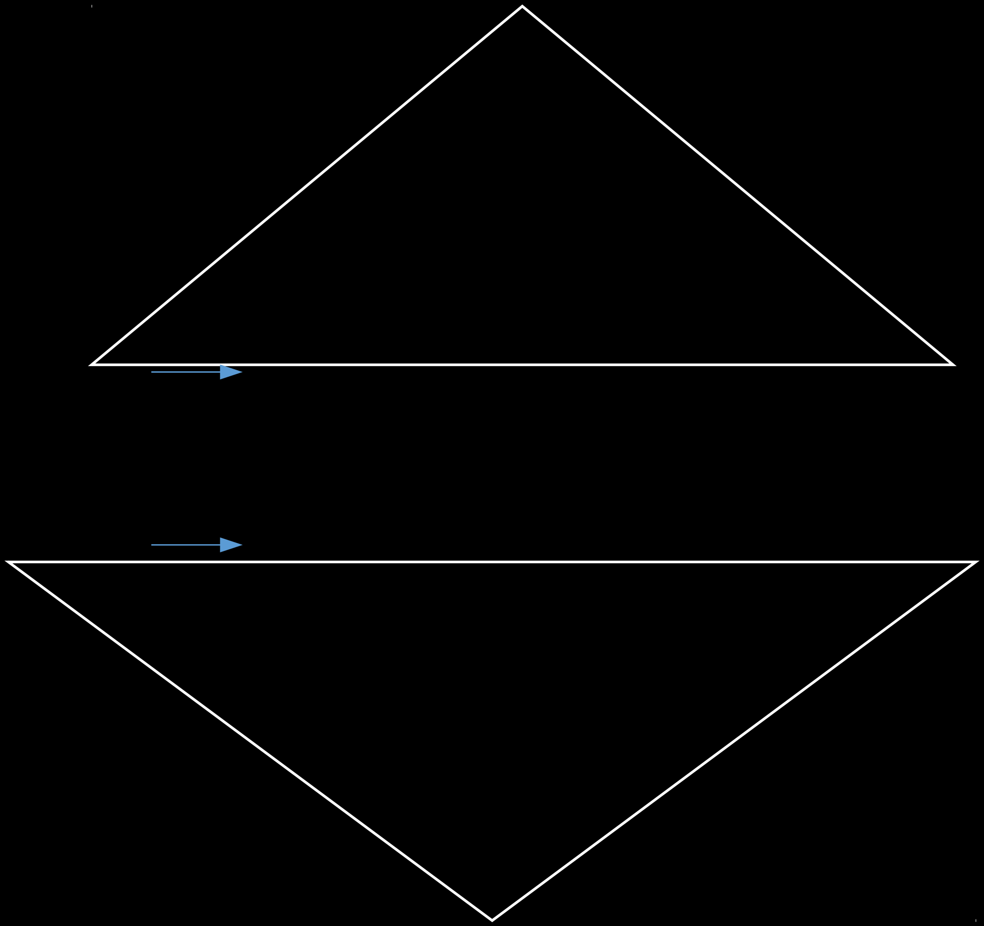


LCE queries



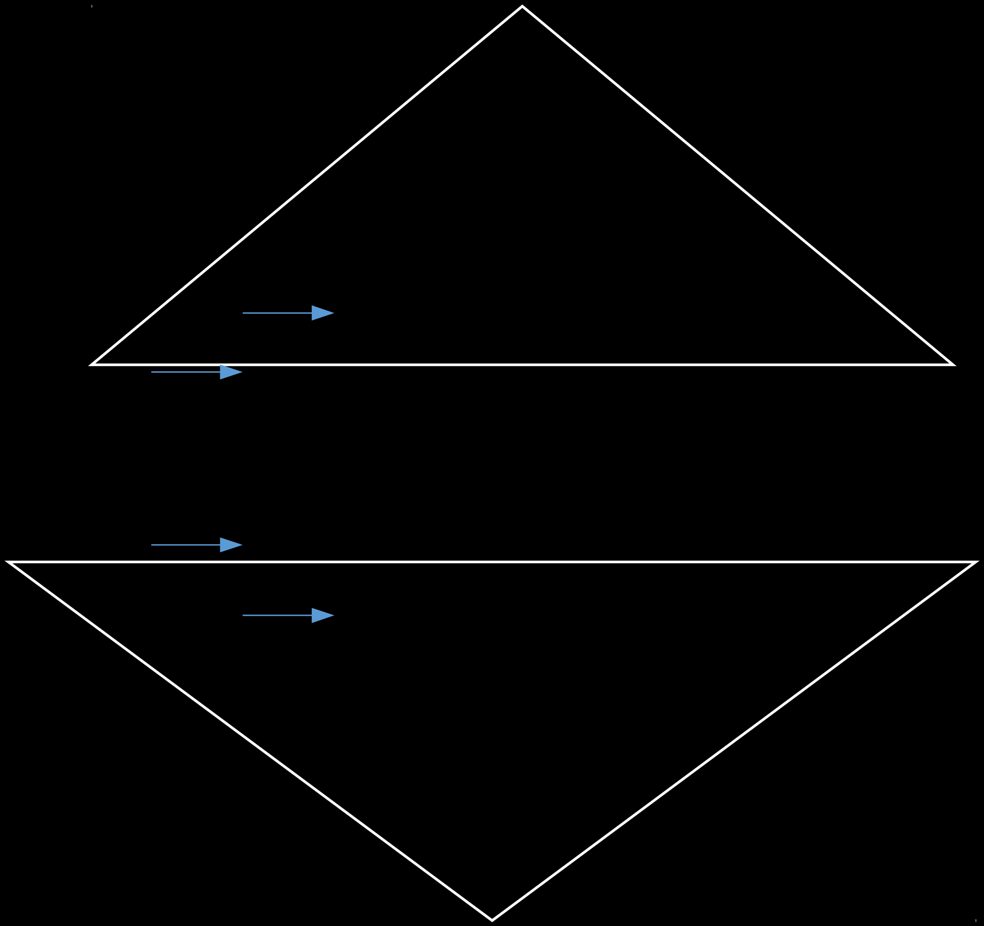
LCE queries

- compare $O(\lg^* n)$ chars



LCE queries

- compare $O(\lg^* n)$ chars
- move upwards
- compare $O(\lg^* n)$ IDs



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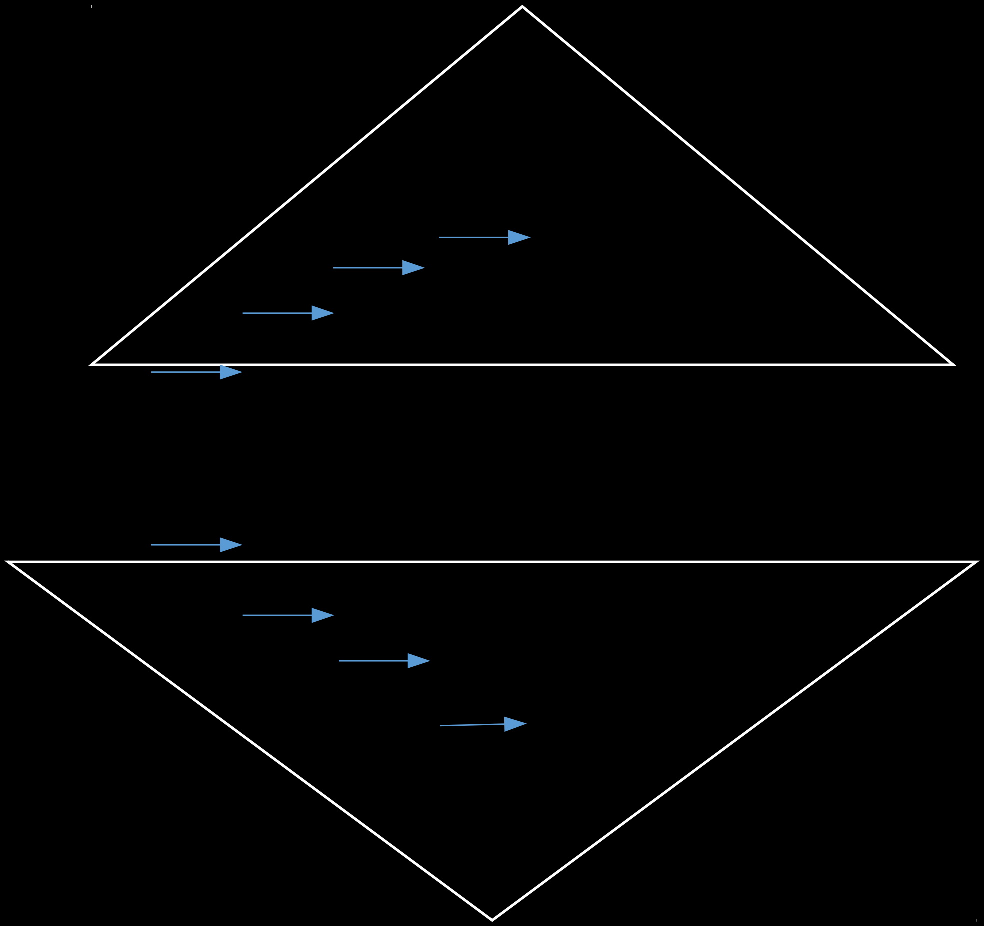

LCE queries

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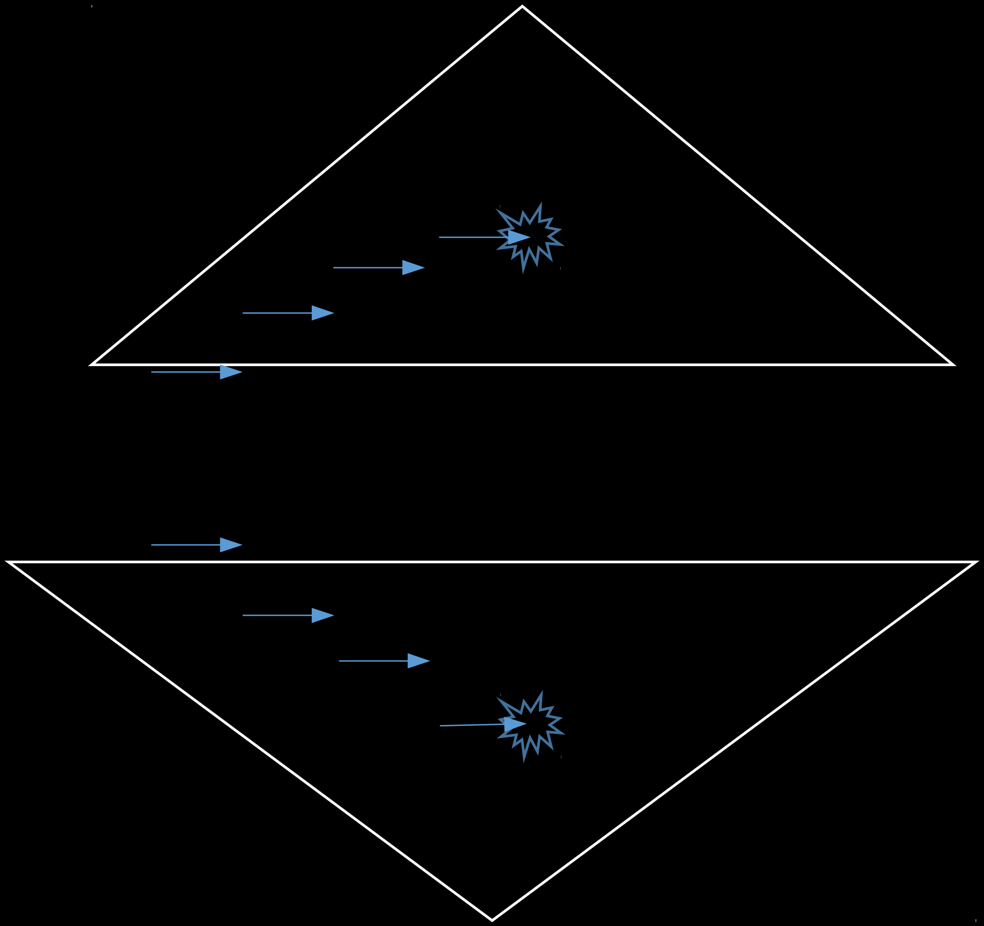
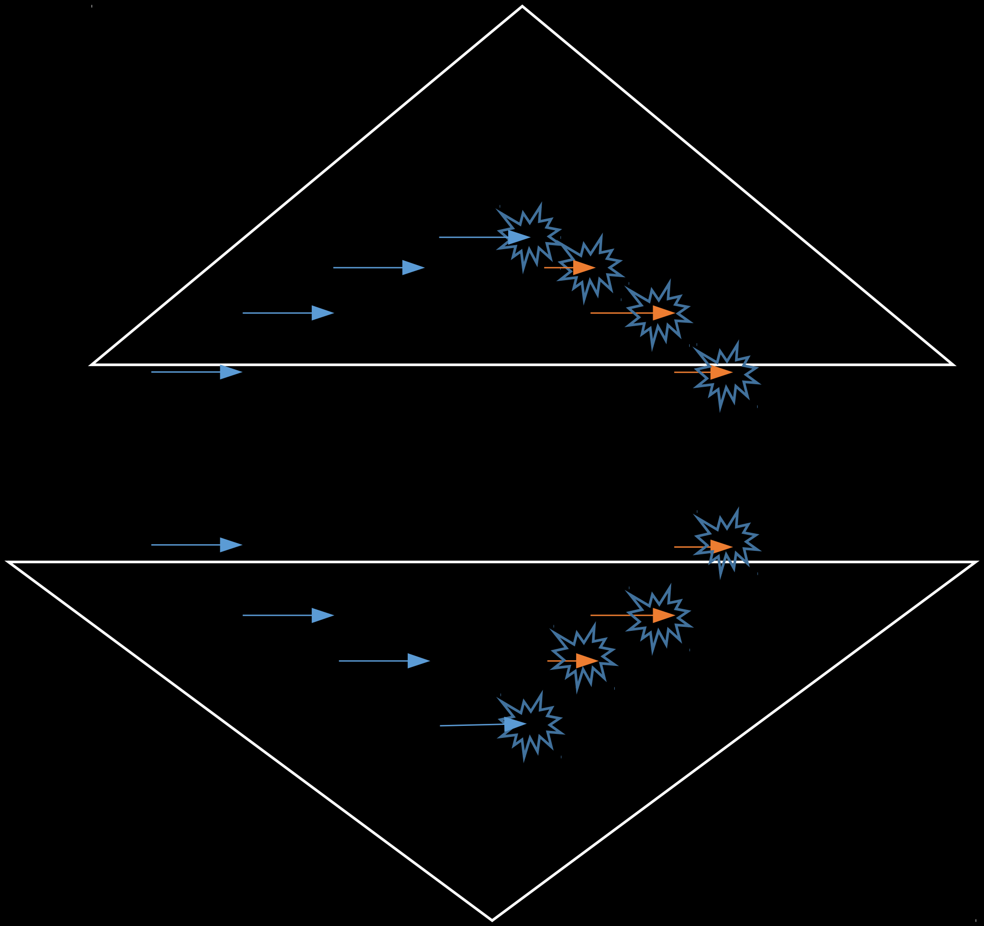
- move upwards
- compare $O(\lg^* n)$ IDs
- (recursion)
- on mismatch:



LCE queries

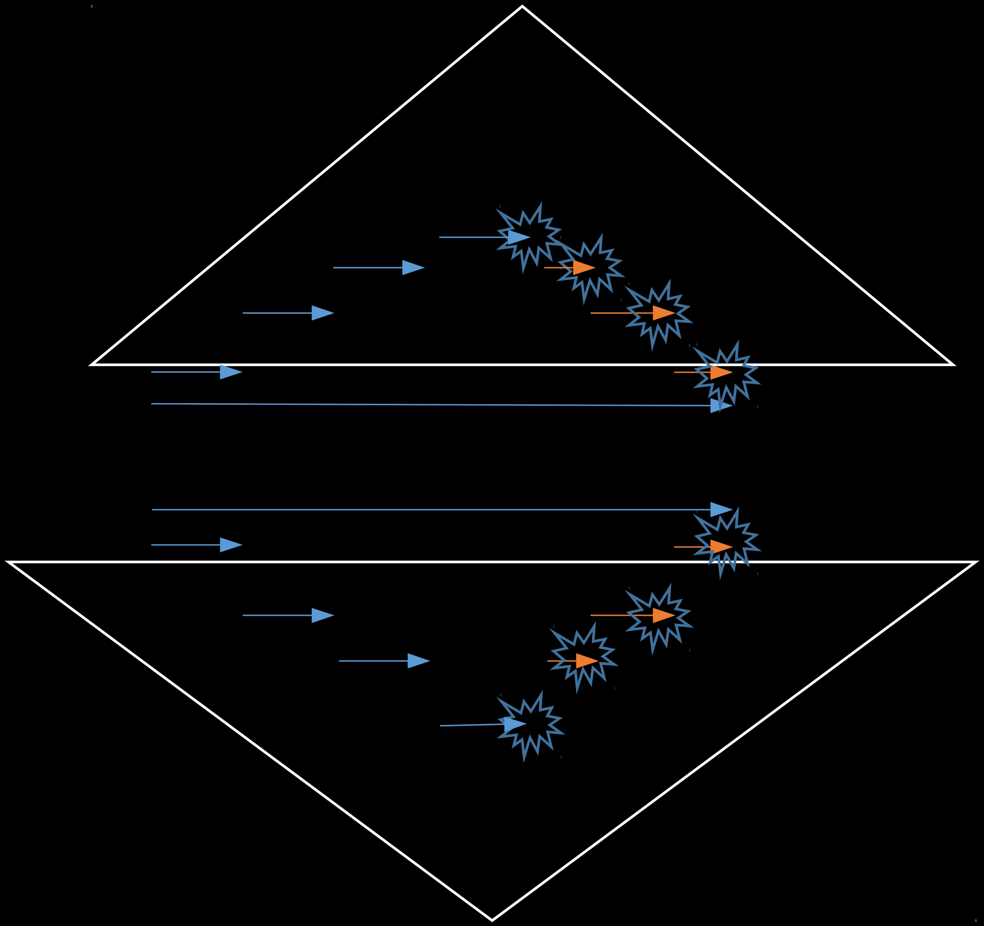
- compare $O(\lg^* n)$ chars
- move upwards
- compare $O(\lg^* n)$ IDs
- (recursion)
- on mismatch:
    - move downwards
    - compare children

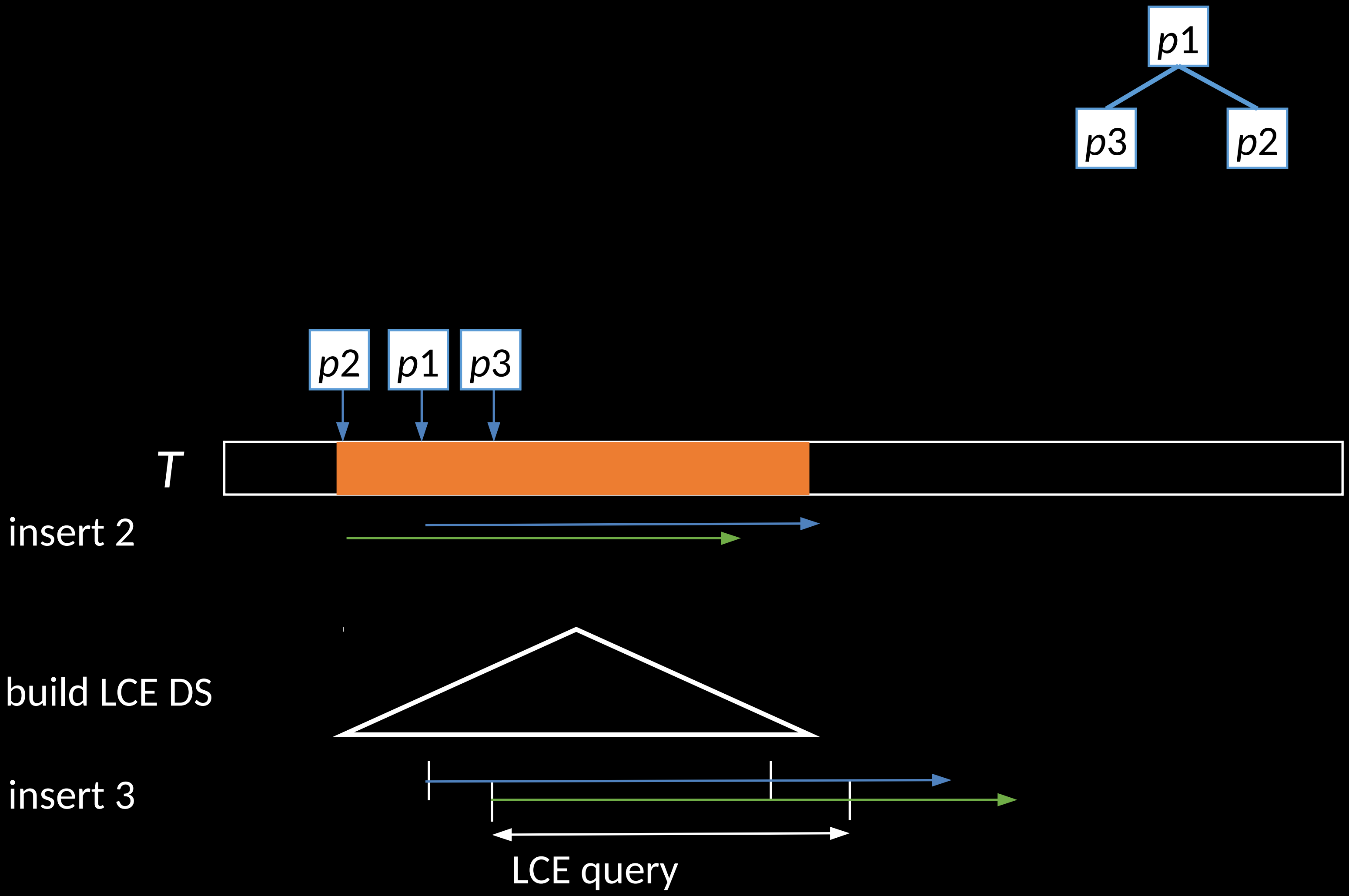


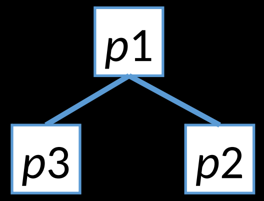
LCE queries

- compare $O(\lg^* n)$ chars
- move upwards
- compare $O(\lg^* n)$ IDs
- (recursion)
- on mismatch:
    - move downwards
    - compare children

$\Rightarrow O(\lg n \lg^* n)$ time



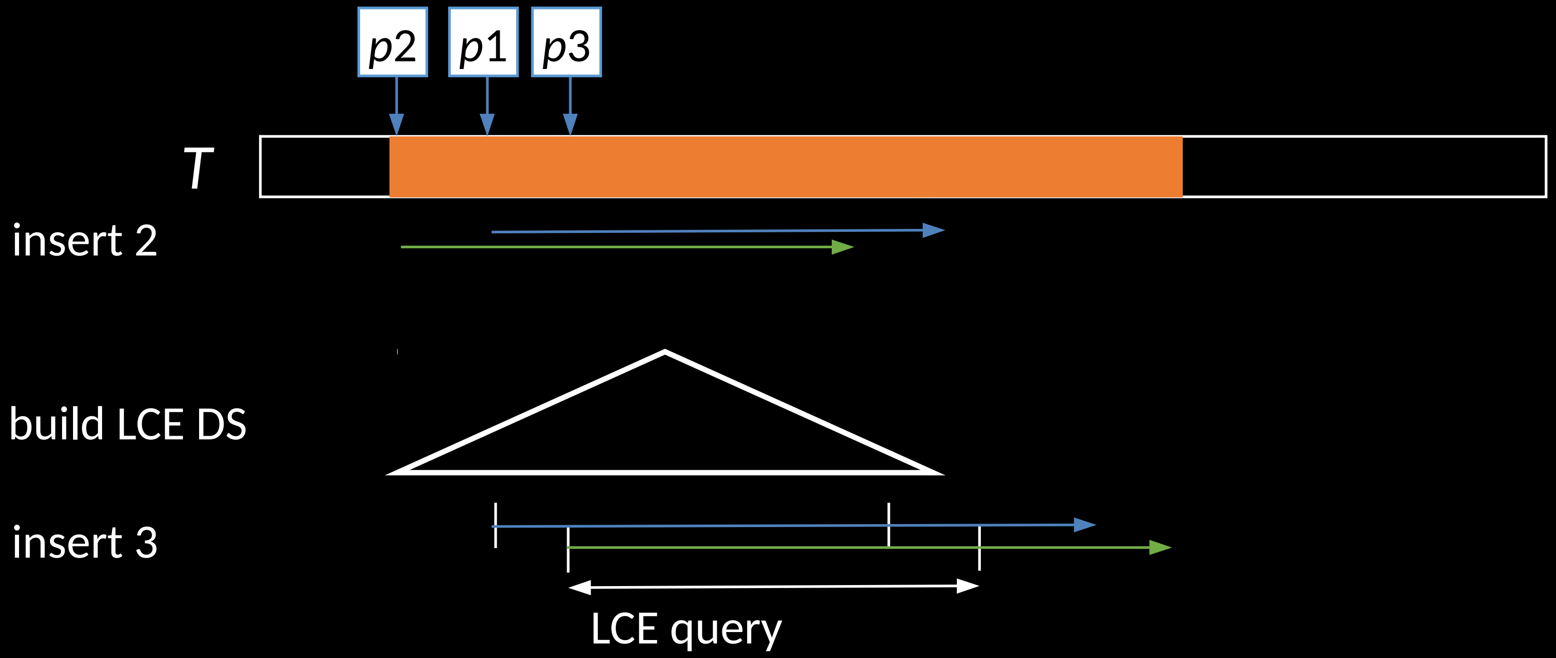

p1
p3
p2
p2
p1
p3
T
insert 2
build LCE DS
insert 3
LCE query



*p*1

*p*3

*p*2



*p*2 *p*1 *p*3

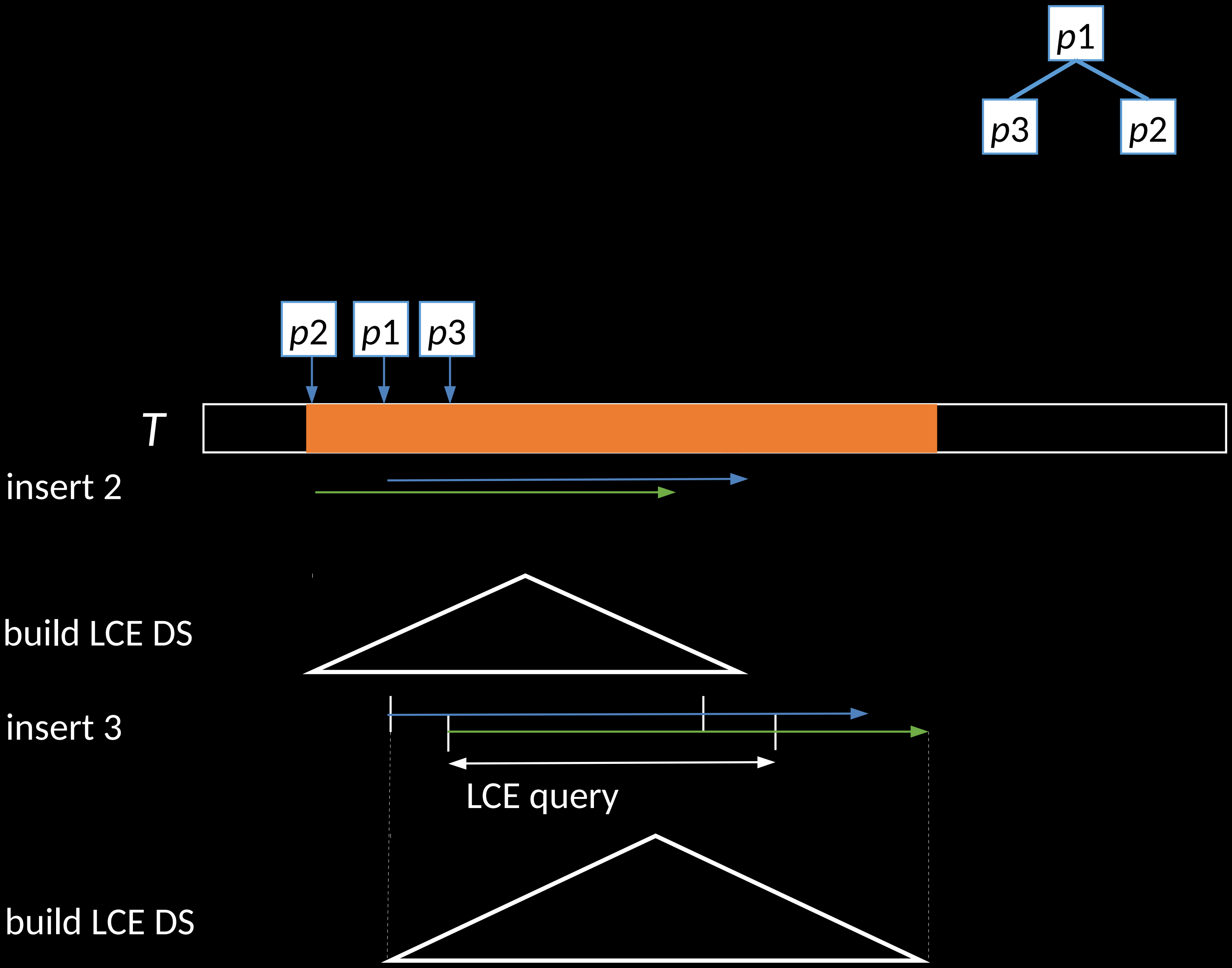
T

insert 2

build LCE DS

insert 3

LCE query



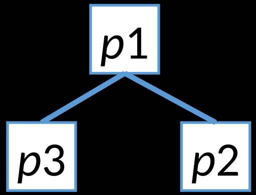
insert 2

build LCE DS

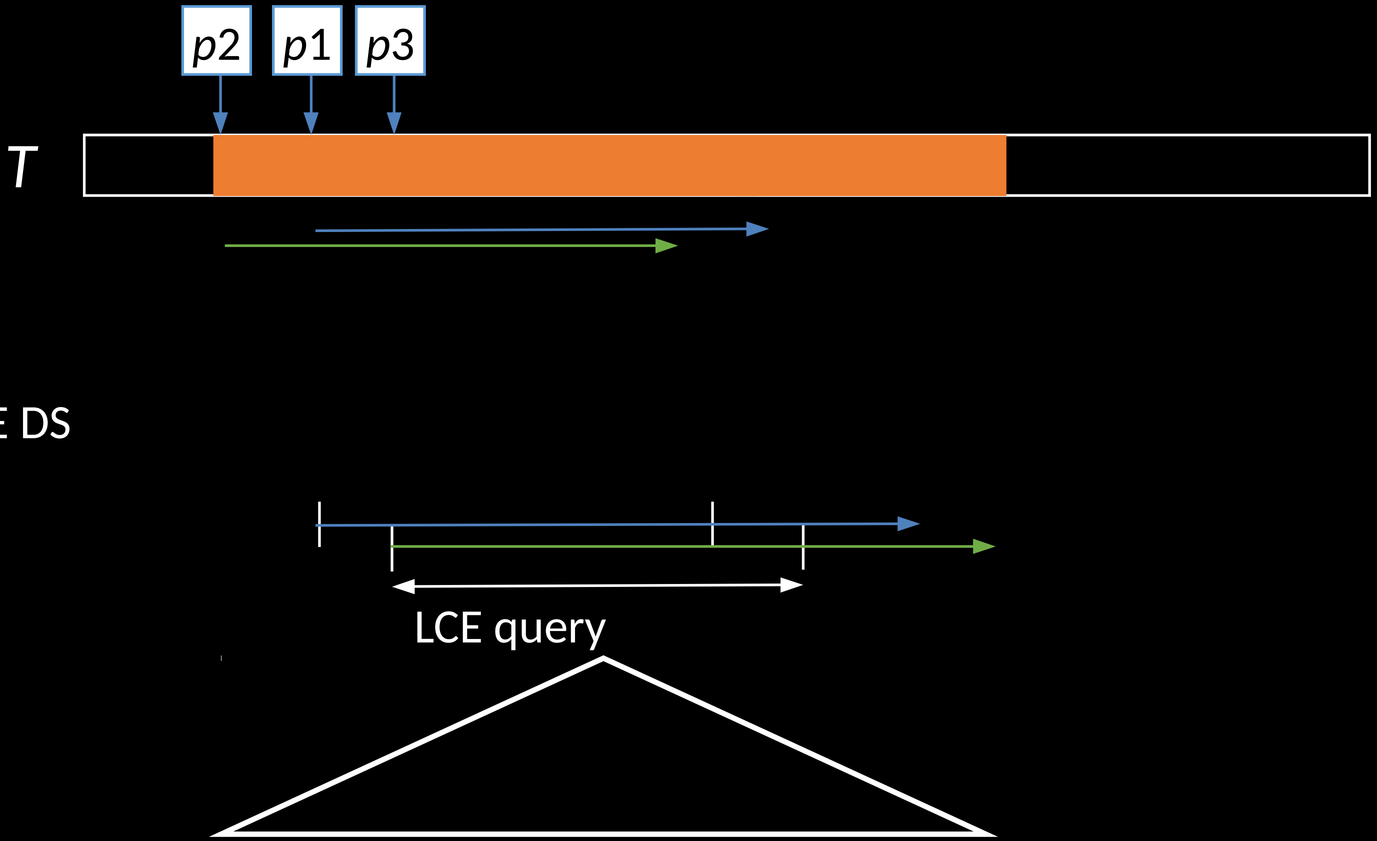
insert 3

LCE query

build LCE DS



how to update?



insert 2

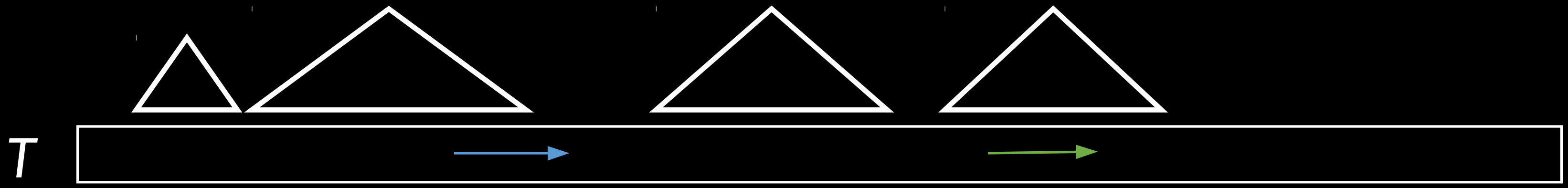
build LCE DS

insert 3

update

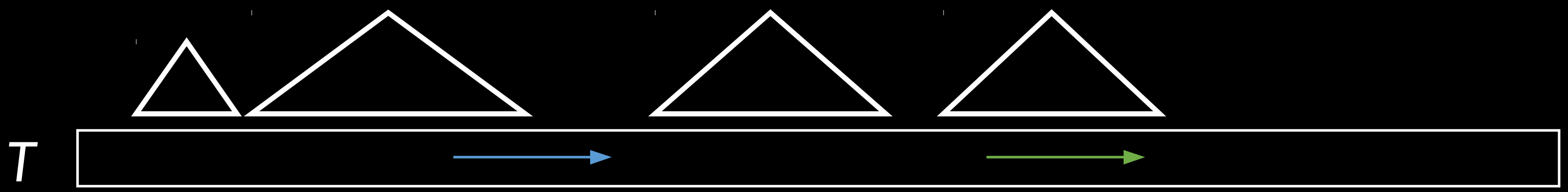
update

- blue arrow exceeds tree‘s range not much
  ⇒ do nothing



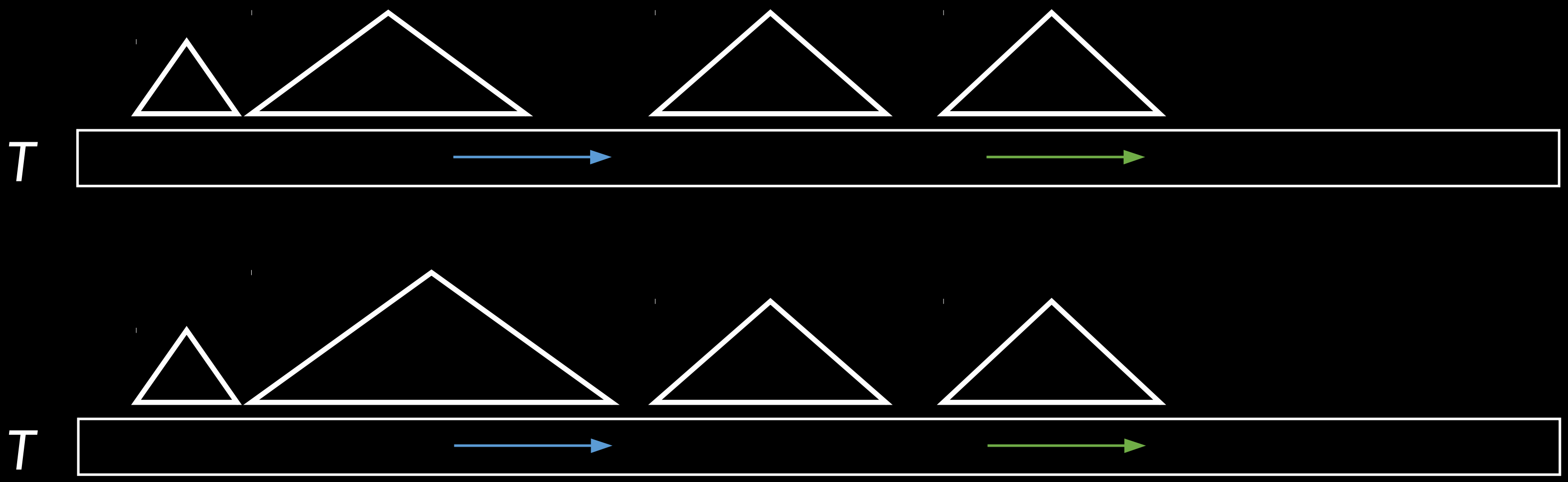
update

- blue arrow exceeds tree‘s range far



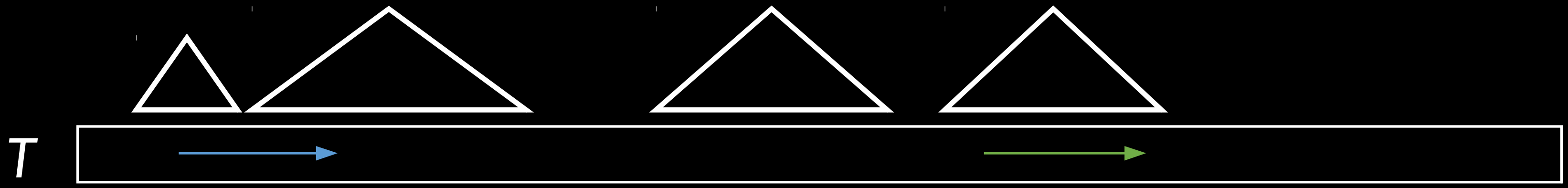
update

- blue arrow exceeds tree‘s range far
  ⇒ enlarge



update

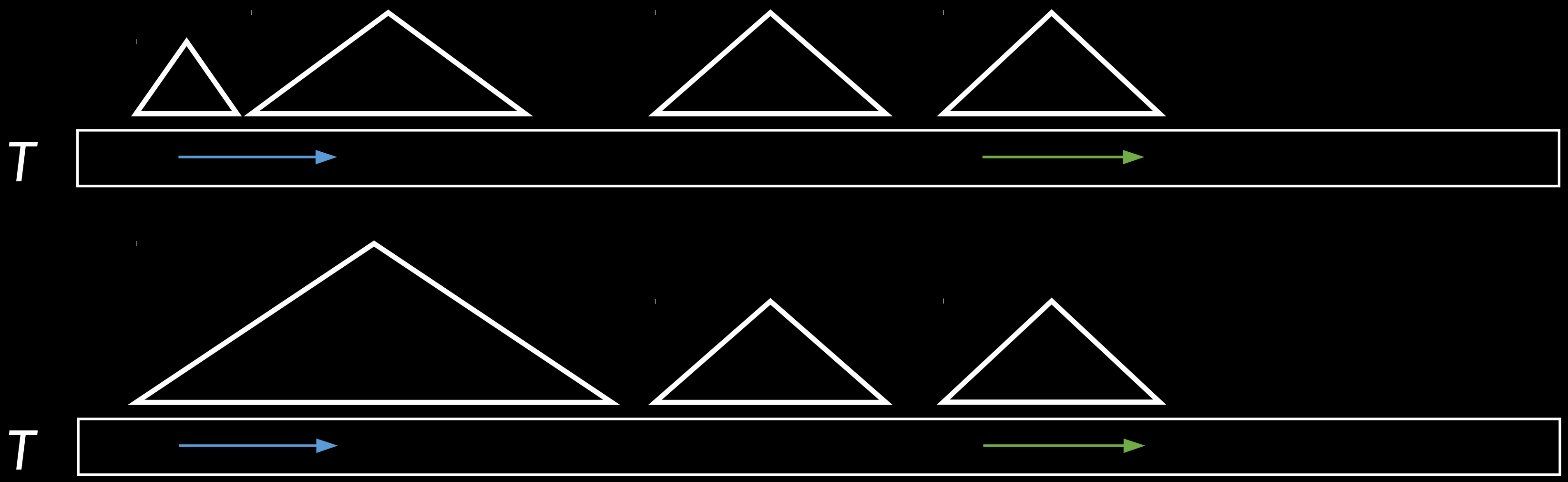
- blue arrow traverses two trees



update

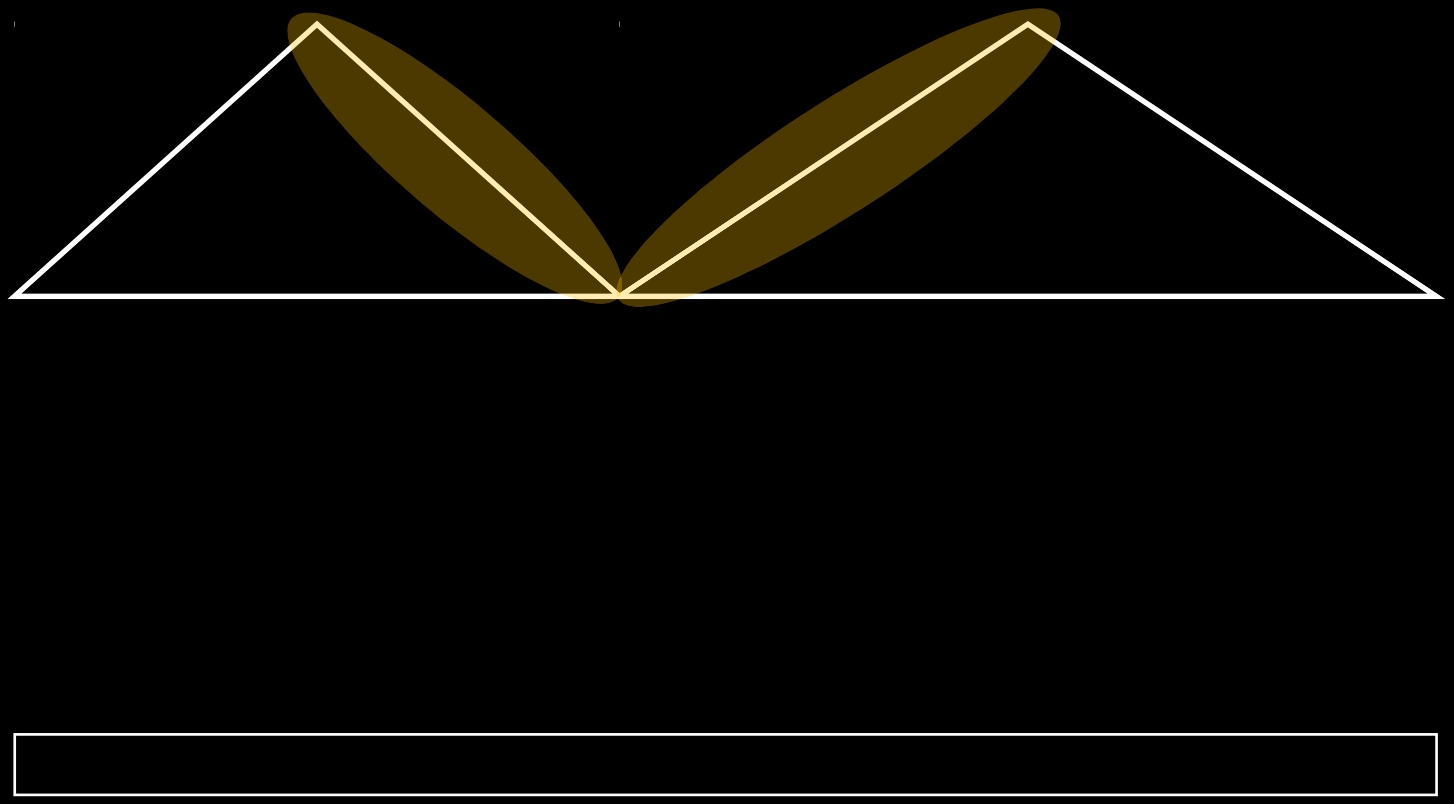
- blue arrow traverses two trees

  ⇒ merge trees



merge

- recalculate local surrounding of nodes near merge



merge

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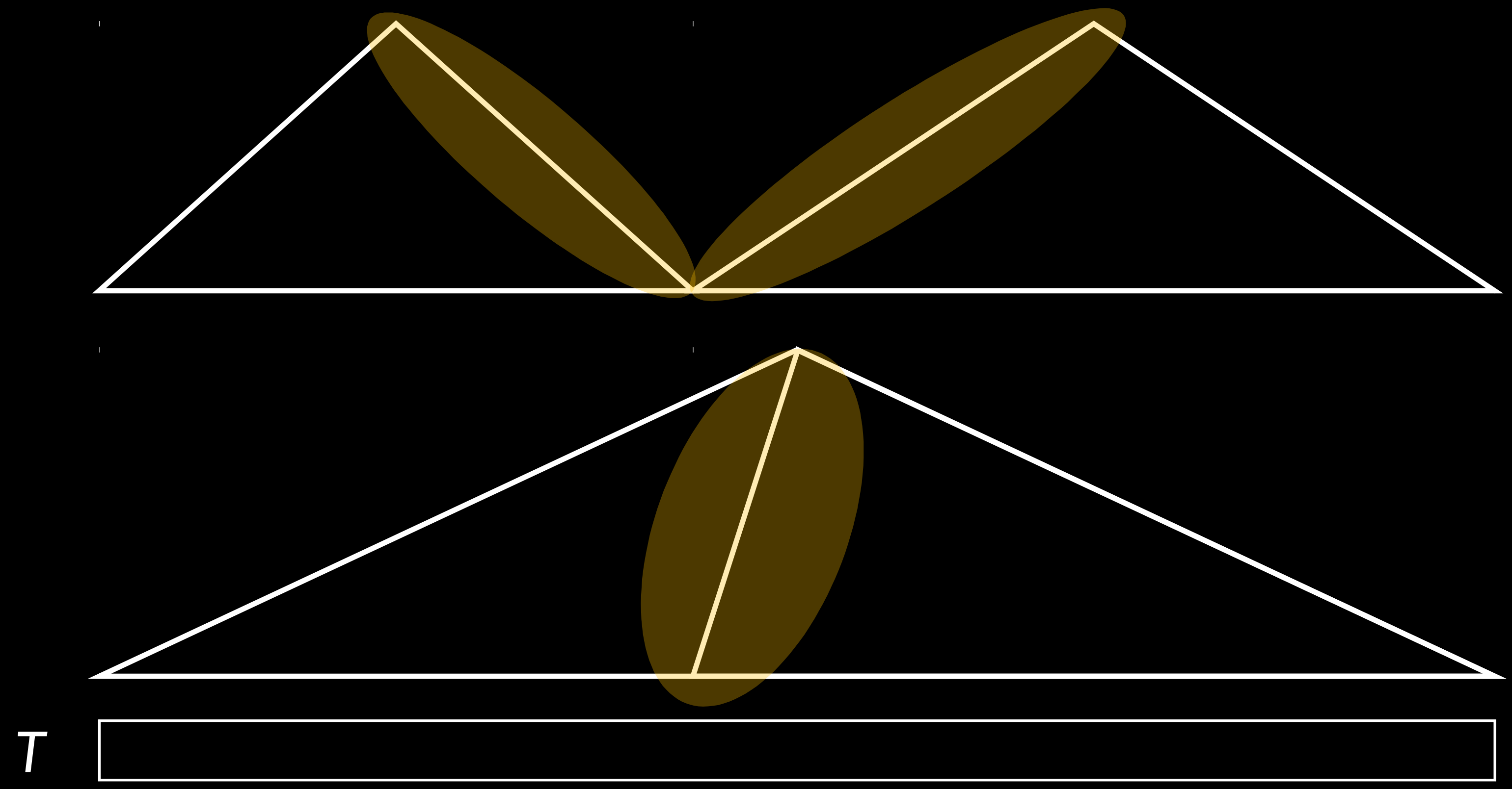
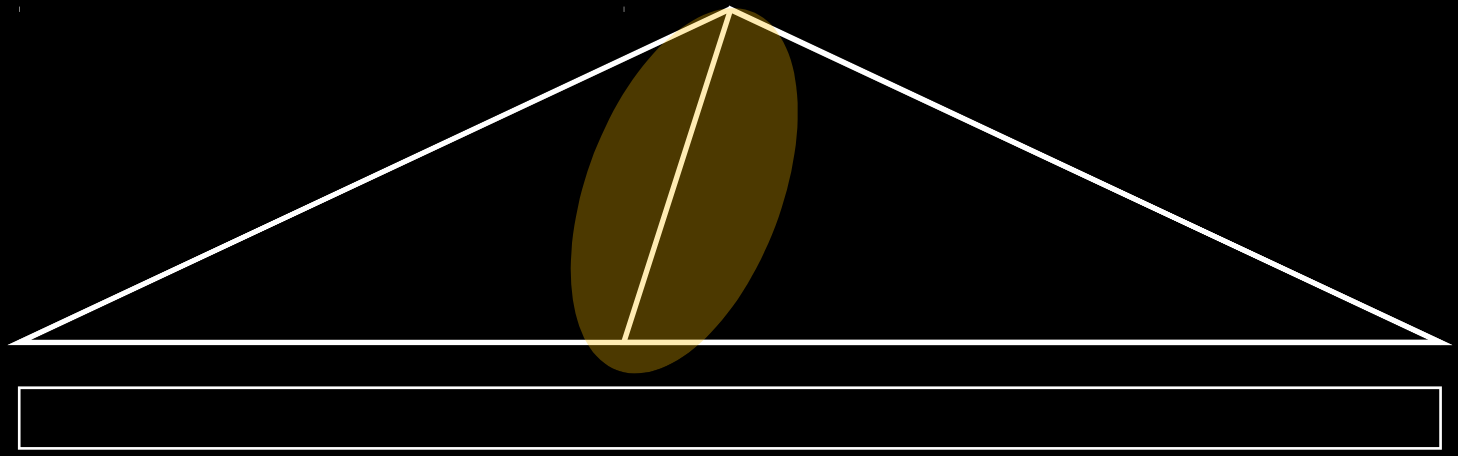


merge

- recalculate local surrounding of nodes near merge
- look at $O(\lg^* n)$ nodes on each height
- dictionary: $O(\lg n)$

local surrounding

$\Rightarrow$ takes $O(\lg n \lg^* n)$ time



total time

- create suffix binary search tree (BST)
- $O(m \lg m)$ time: put m suffixes in BST
    - $O(\lg n \lg^* n)$ time: LCE query
- $O(c \lg n)$ time: building ESP trees
- $O(m \lg m \lg n \lg^* n)$ time for merging
- total:

$$O(\underbrace{m \lg m}_{\text{BST}} \overbrace{\lg n \lg^* n}^{\text{LCE}} + \overbrace{c \lg n}^{\text{building}})$$

- if $m, c = o(n) \Rightarrow o(n)$ overall time!

space

- $O(m)$ words for
  - BST sorting suffixes
  - locating ESP trees
- $O(c \lg n)$ bits for
  - ESP trees
  - grammar dictionary

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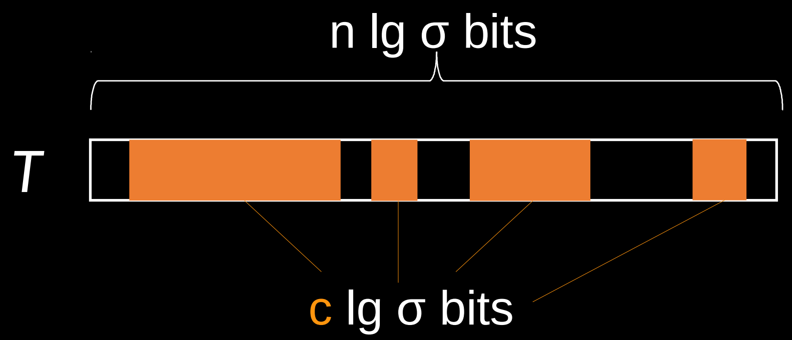


n lg σ bits

T

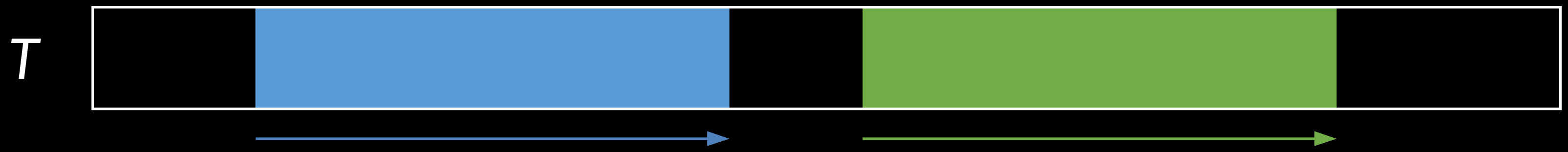
c lg σ bits

how to shrink ESP tree to $O(c \lg \sigma)$ bits?

overwriting text space

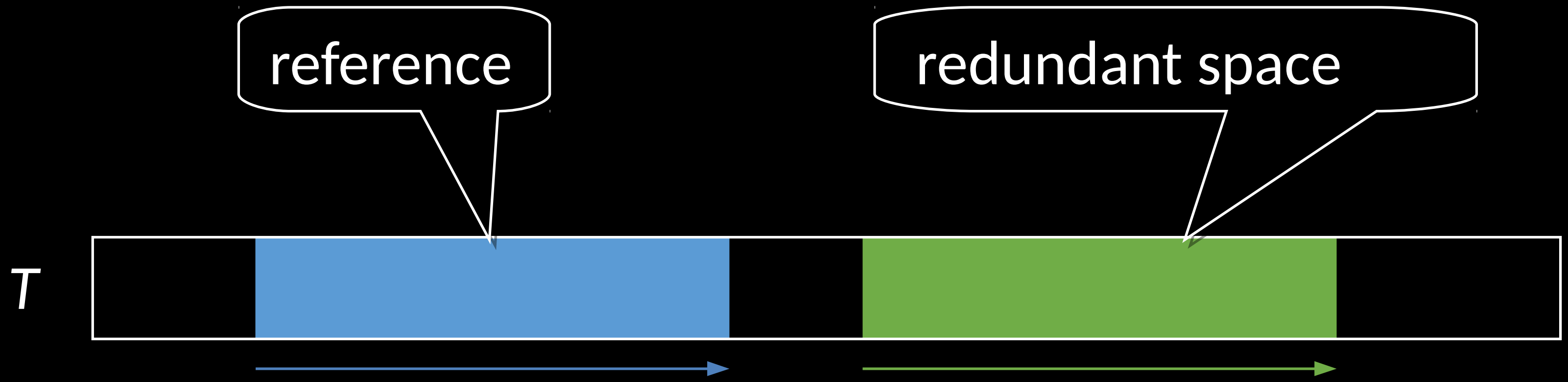
overwriting text space

- on finding two equal substrings



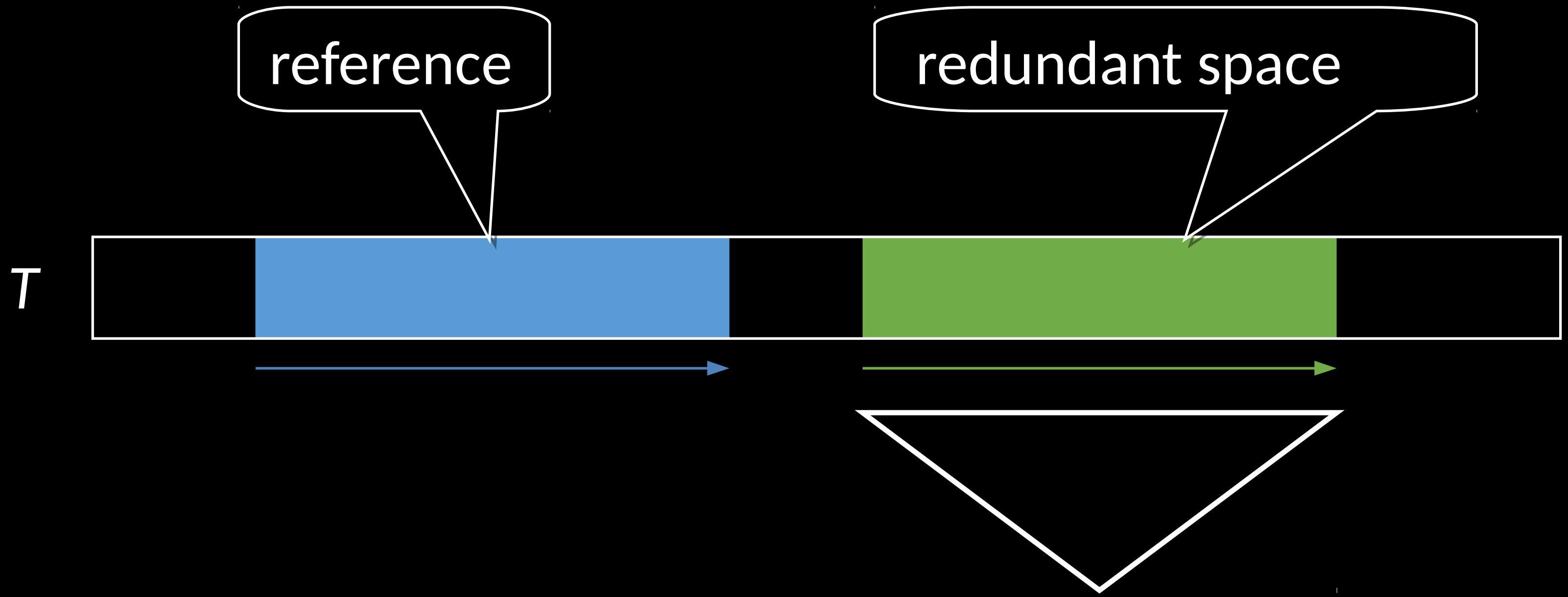
overwriting text space

- on finding two equal substrings



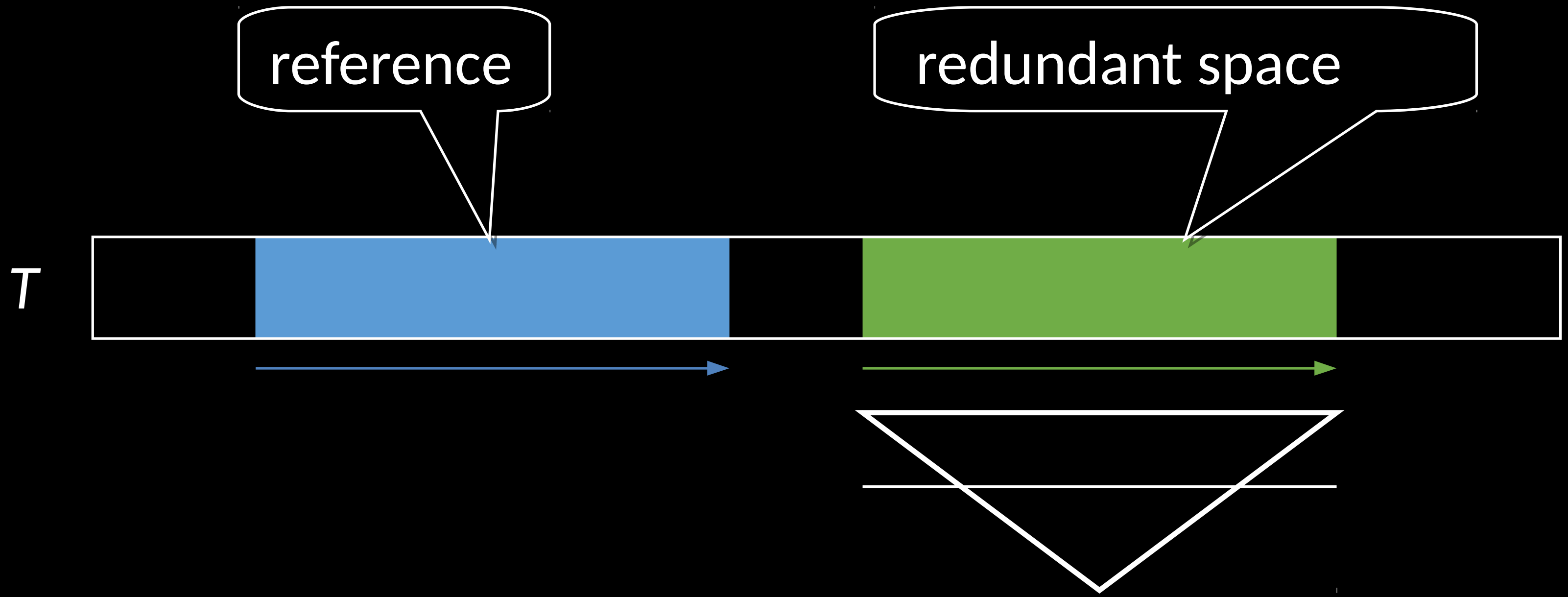
overwriting text space

- on finding two equal substrings
    - build ESP tree in redundant space



overwriting text space

- on finding two equal substrings
  - build ESP tree in redundant space
- truncate tree to fit into text space



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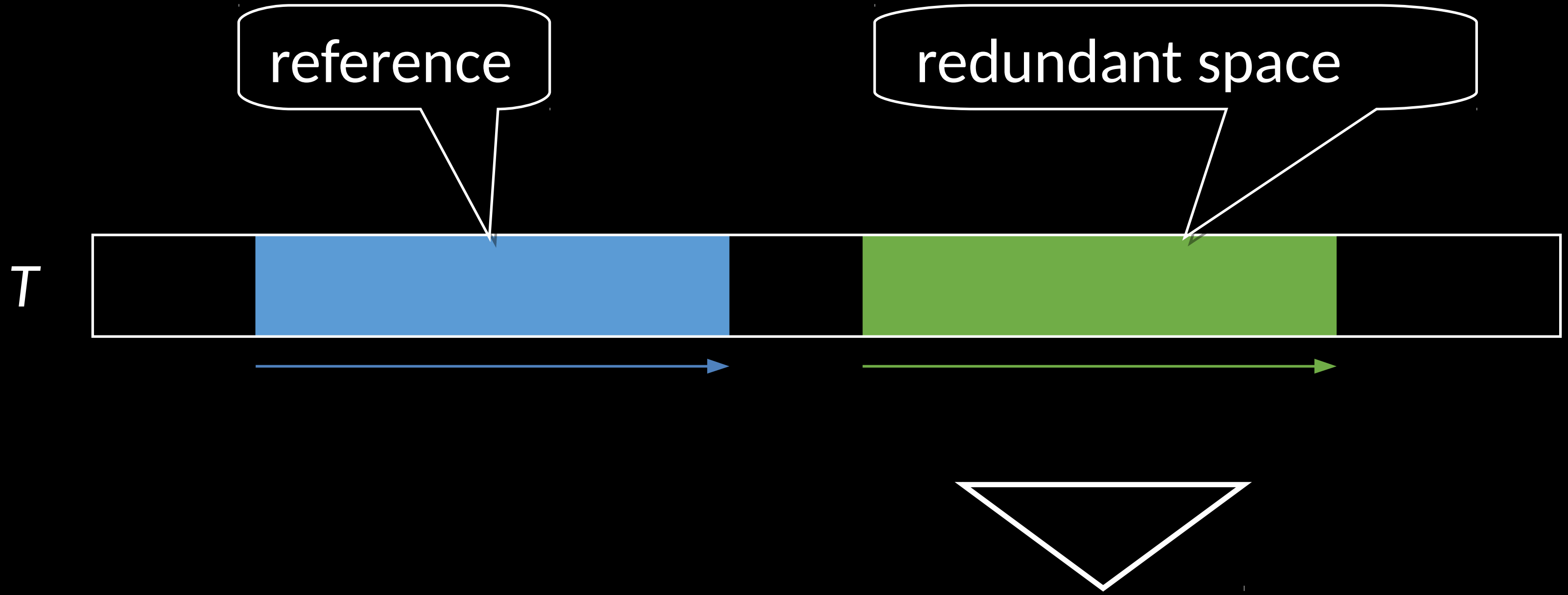

overwriting text space

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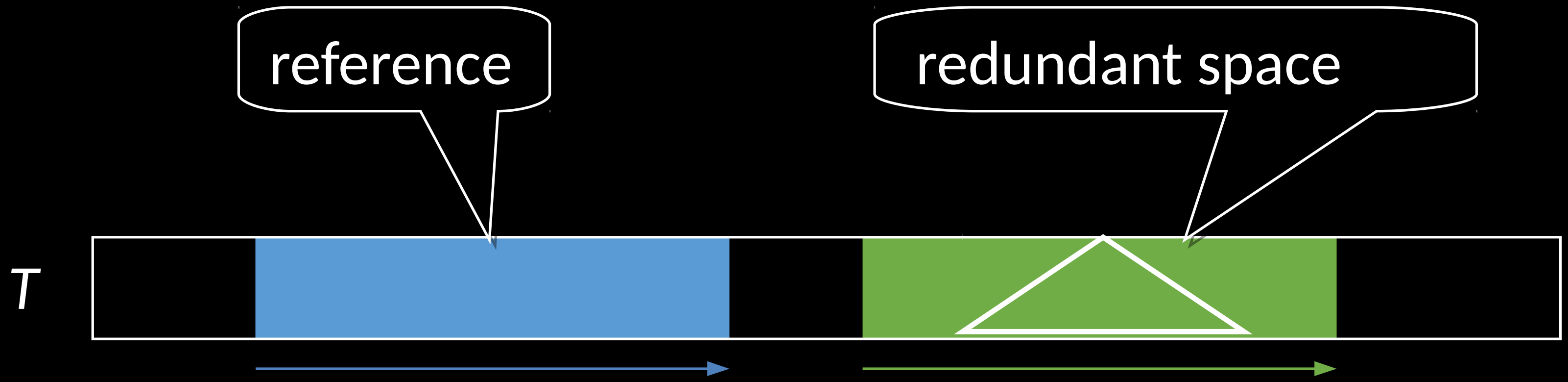

time

- before:

  $O(m \lg m \lg n \lg^* n + c \lg n)$ time

- now: $O(c \lg^{0.5} n)$ time: construction

  $\Rightarrow$ $O(m \lg m \lg n \lg^* n + c \lg^{0.5} n)$ time overall

summary

- sparse suffix sorting problem
  - n: $|T|$
  - m: #positions
- solved with

$O(m \lg m \lg n \lg^* n + c \lg^{0.5} n)$ time
$O(m)$ additional space

- by
  - LCE queries on ESP trees
  - truncation of ESP trees
  - sophisticated merging

summary

- sparse suffix sorting problem
  - n: $|T|$
  - m: #positions
- solved with

$$\boxed{\begin{array}{l} O(m \lg m \lg n \lg^* n + c \lg^{0.5} n) \text{ time} \\ O(m) \text{ additional space} \end{array}}$$

- by
  - LCE queries on ESP trees
  - truncation of ESP trees
  - sophisticated merging

that is all - any questions?