technische universität
dortmund

Practical Evaluation of LZ78 and LZW Tries

Johannes Fischer

Dominik Köppl

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LZ78 with LZ trie

senescence

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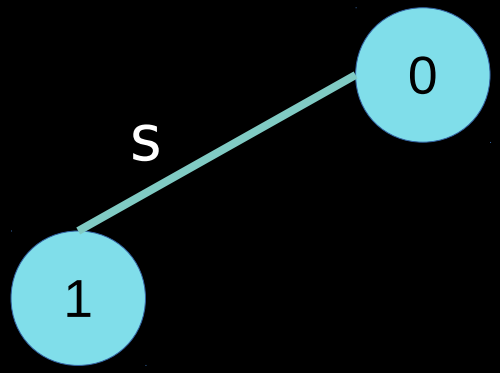
Senescence



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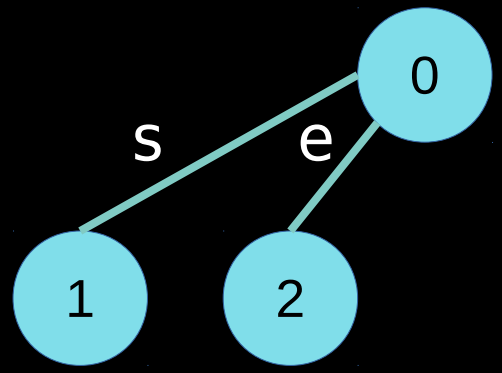

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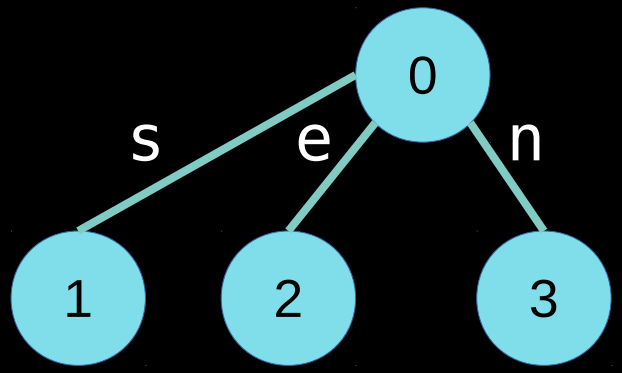

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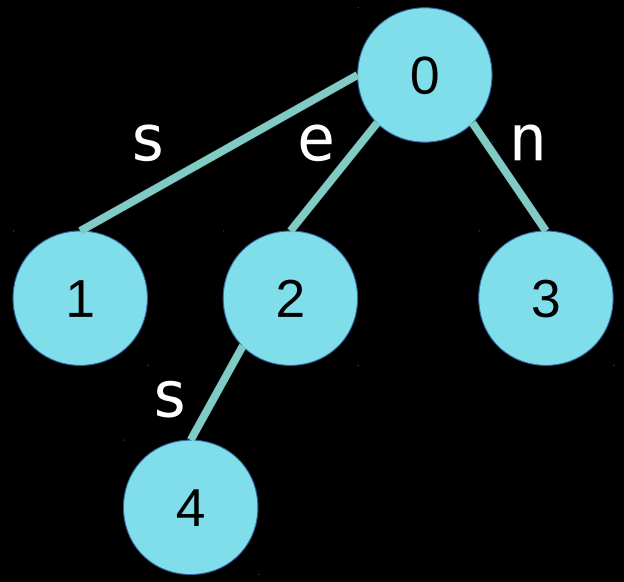

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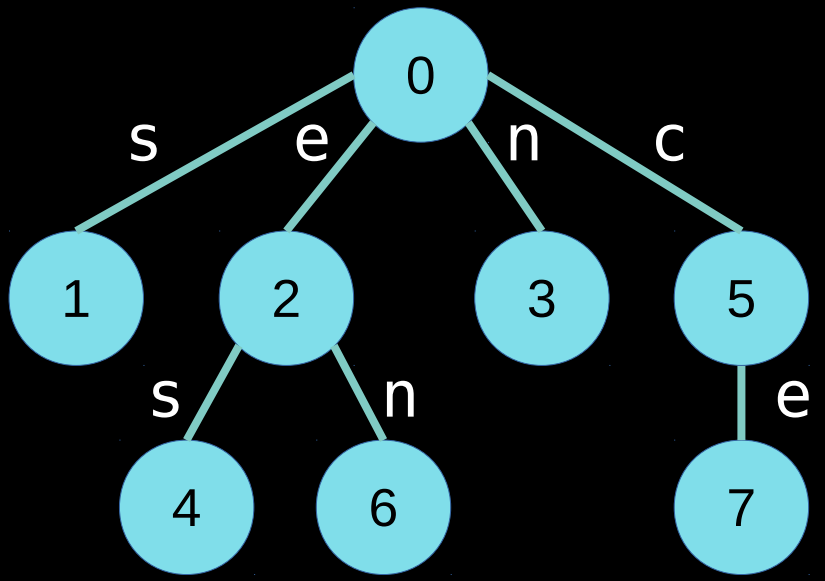

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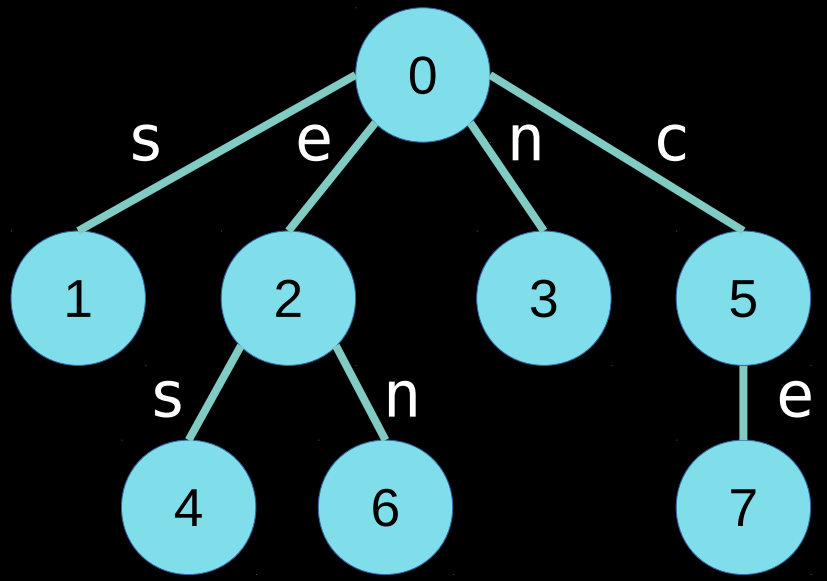

Setting

- in what time / memory build LZ trie *practically*?
- σ: alphabet size
- z: number of factors

trie representations

- binary trie [folklore]
- ternary trie [Bentley, Sedgewick'97]
- hash trie
- compact hash trie
- rolling hash trie

new

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all use dynamic arrays

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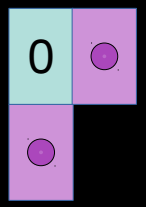
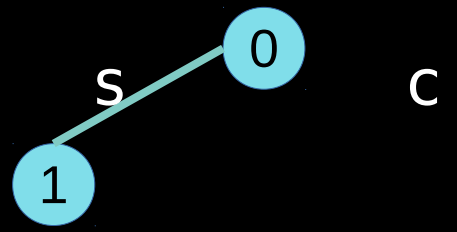
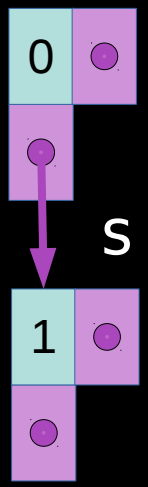


binary trie



Senescence



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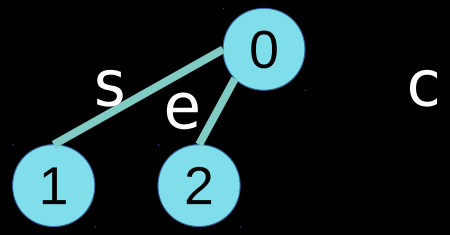

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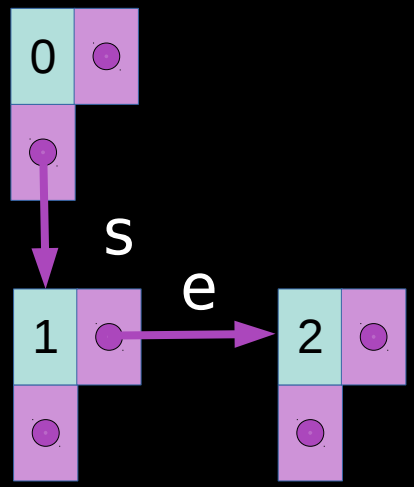

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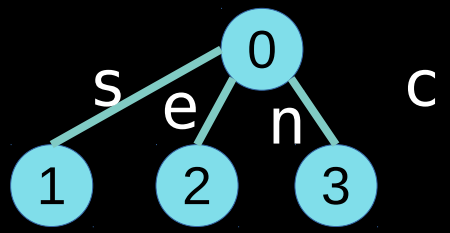

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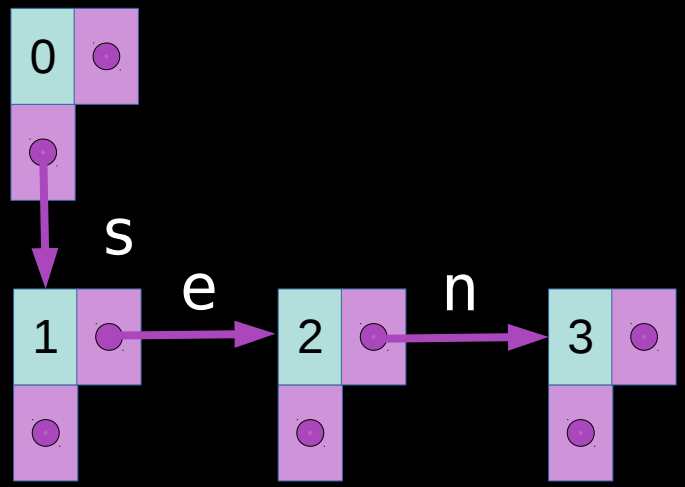
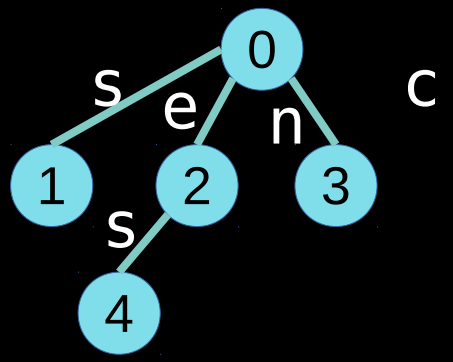


binary trie



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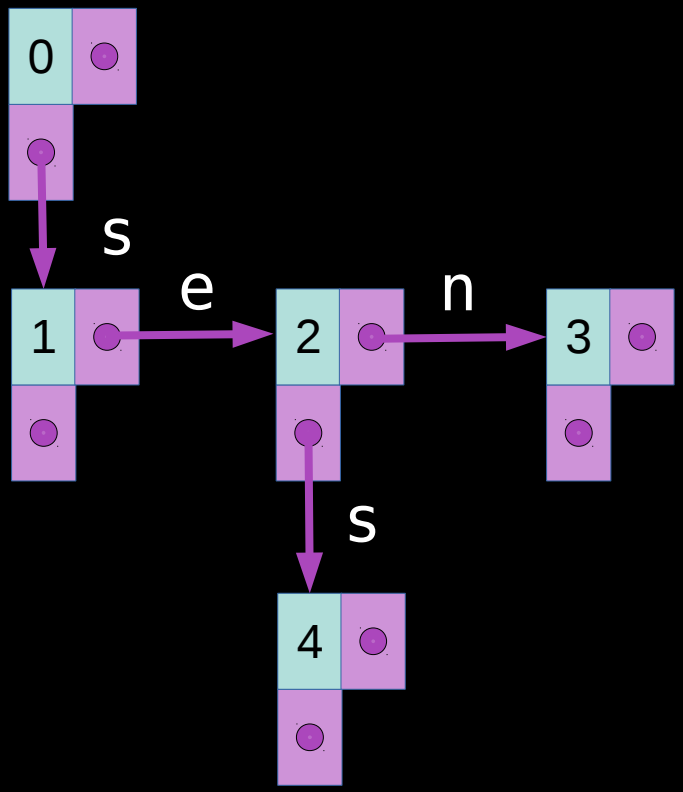

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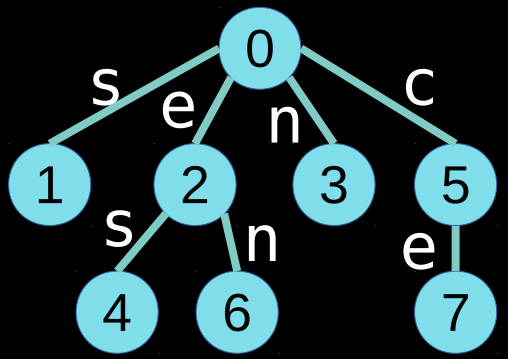

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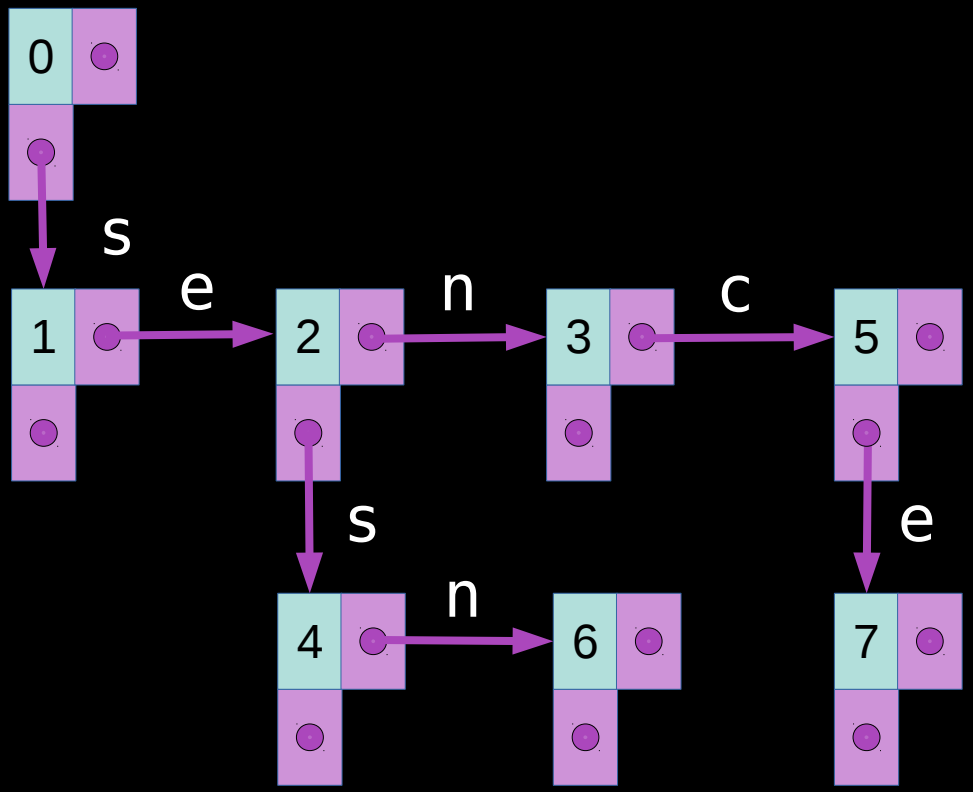
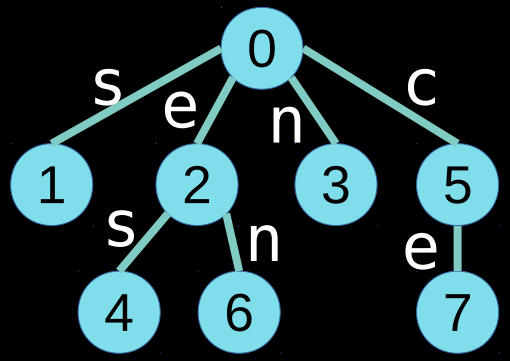


binary trie



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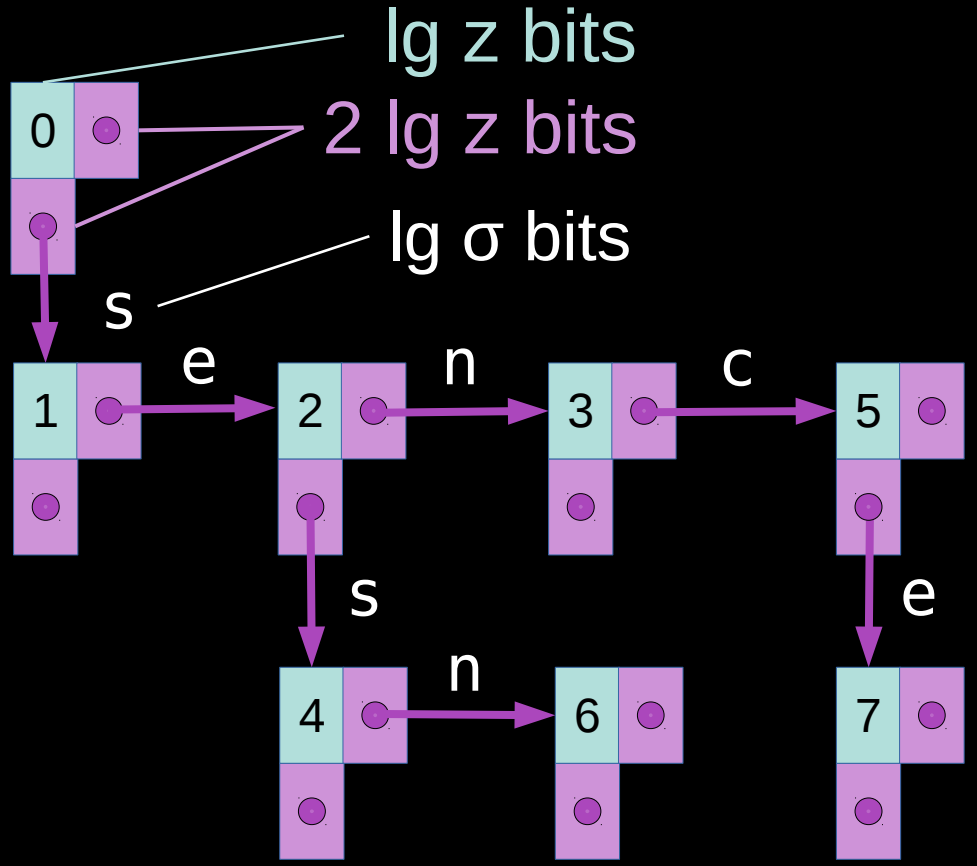
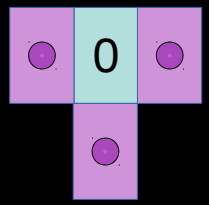


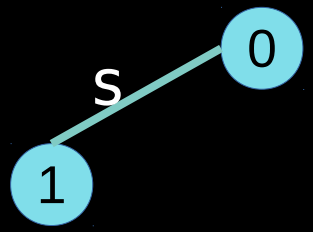
0

ternary trie

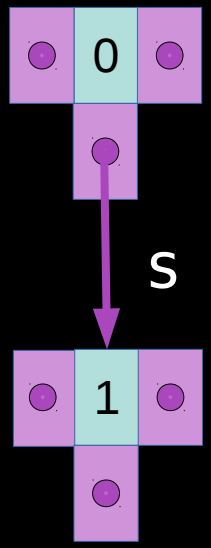
senescence



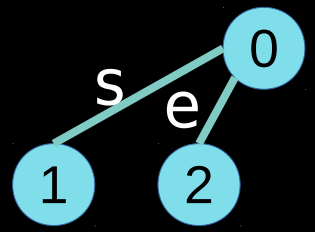
ternary trie



senescence



ternary trie



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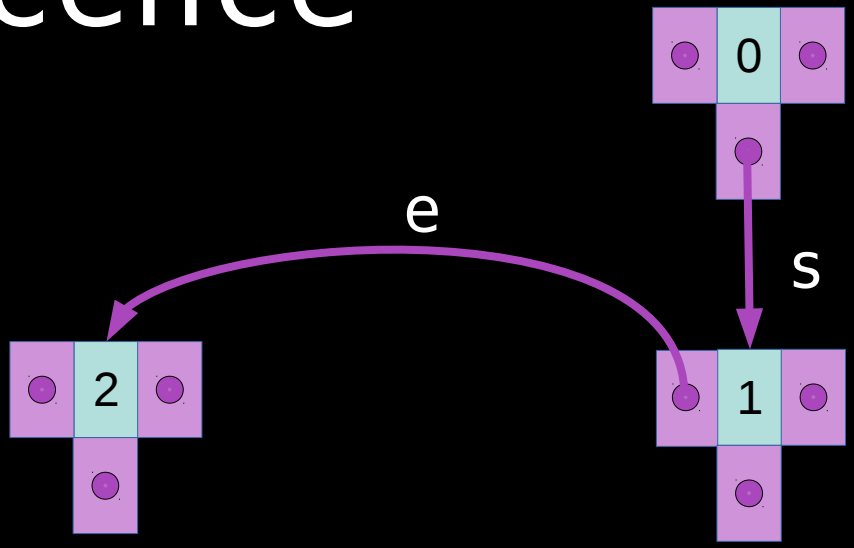
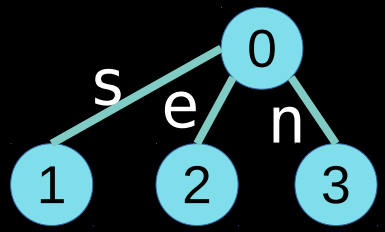


ternary trie



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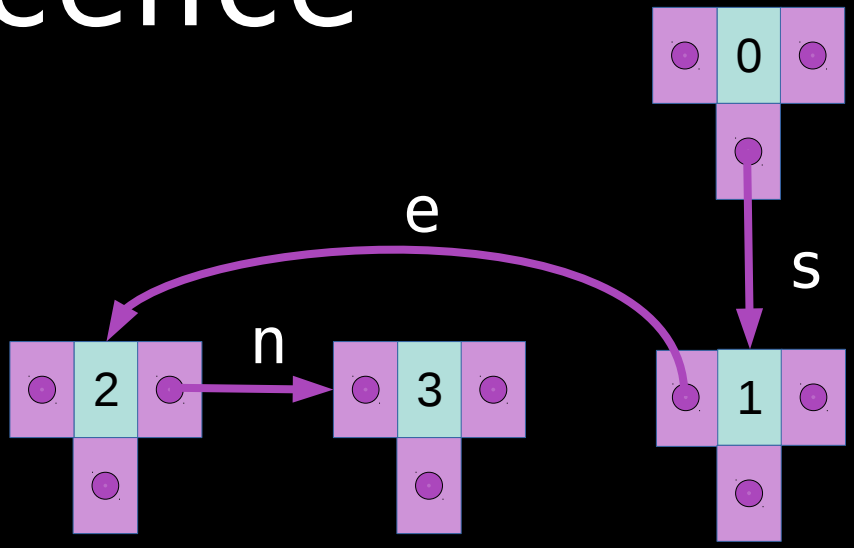
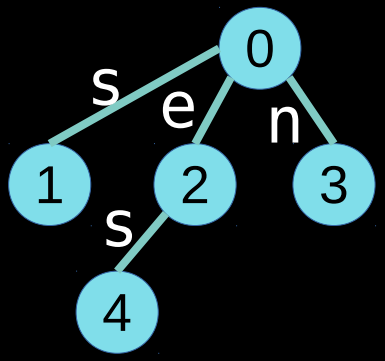


ternary trie



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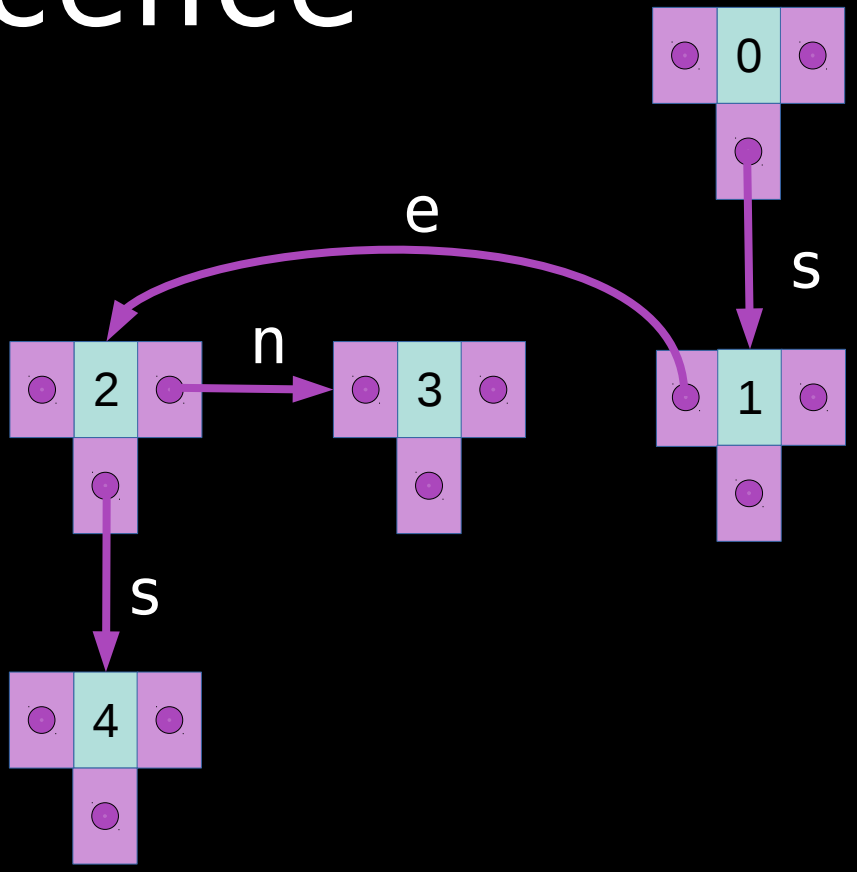

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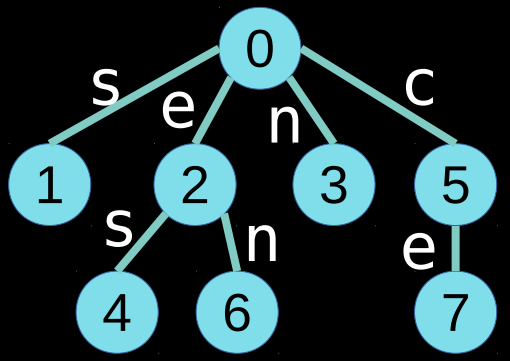

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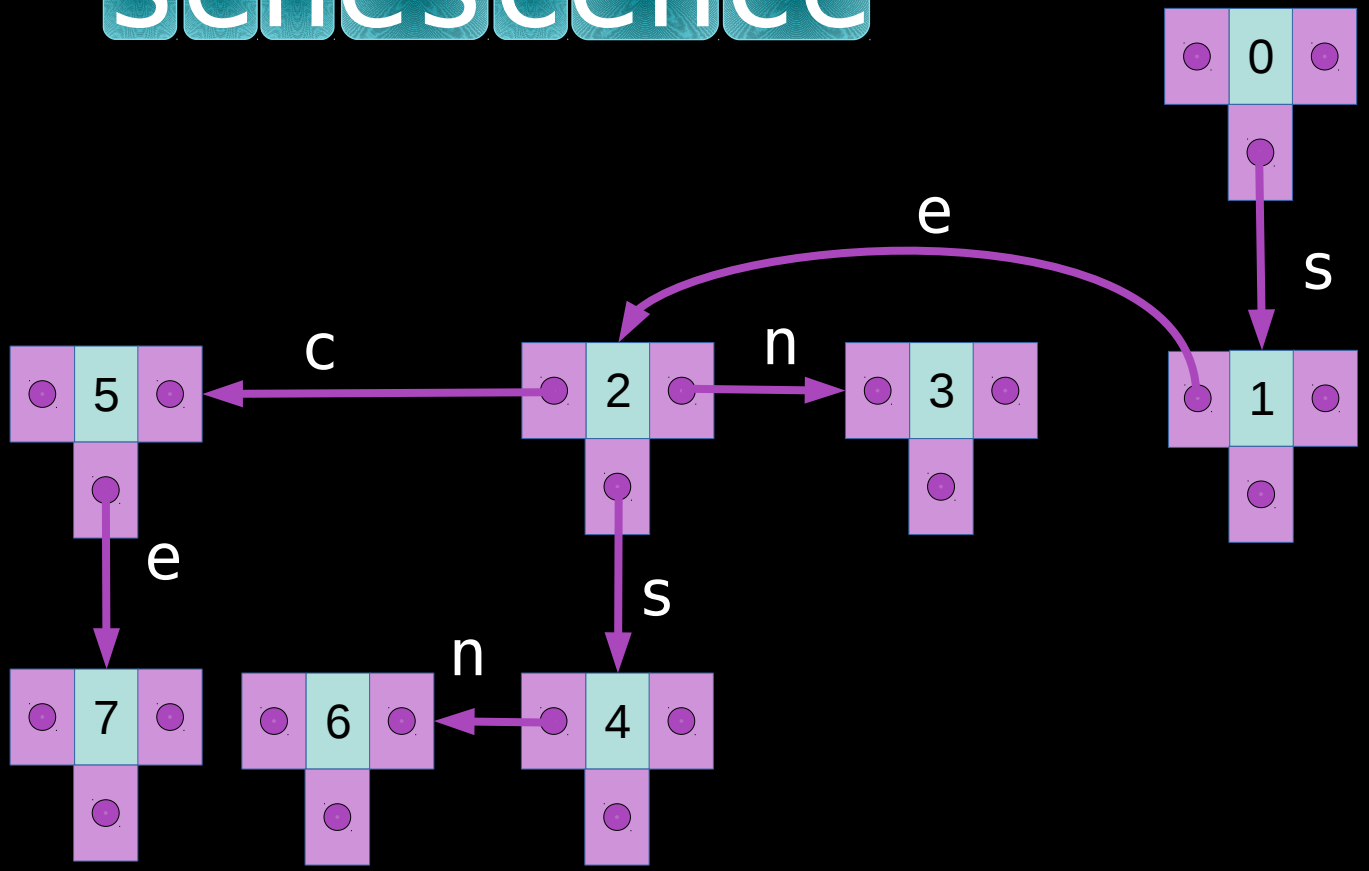

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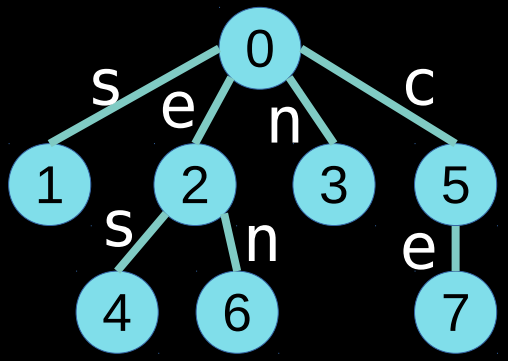

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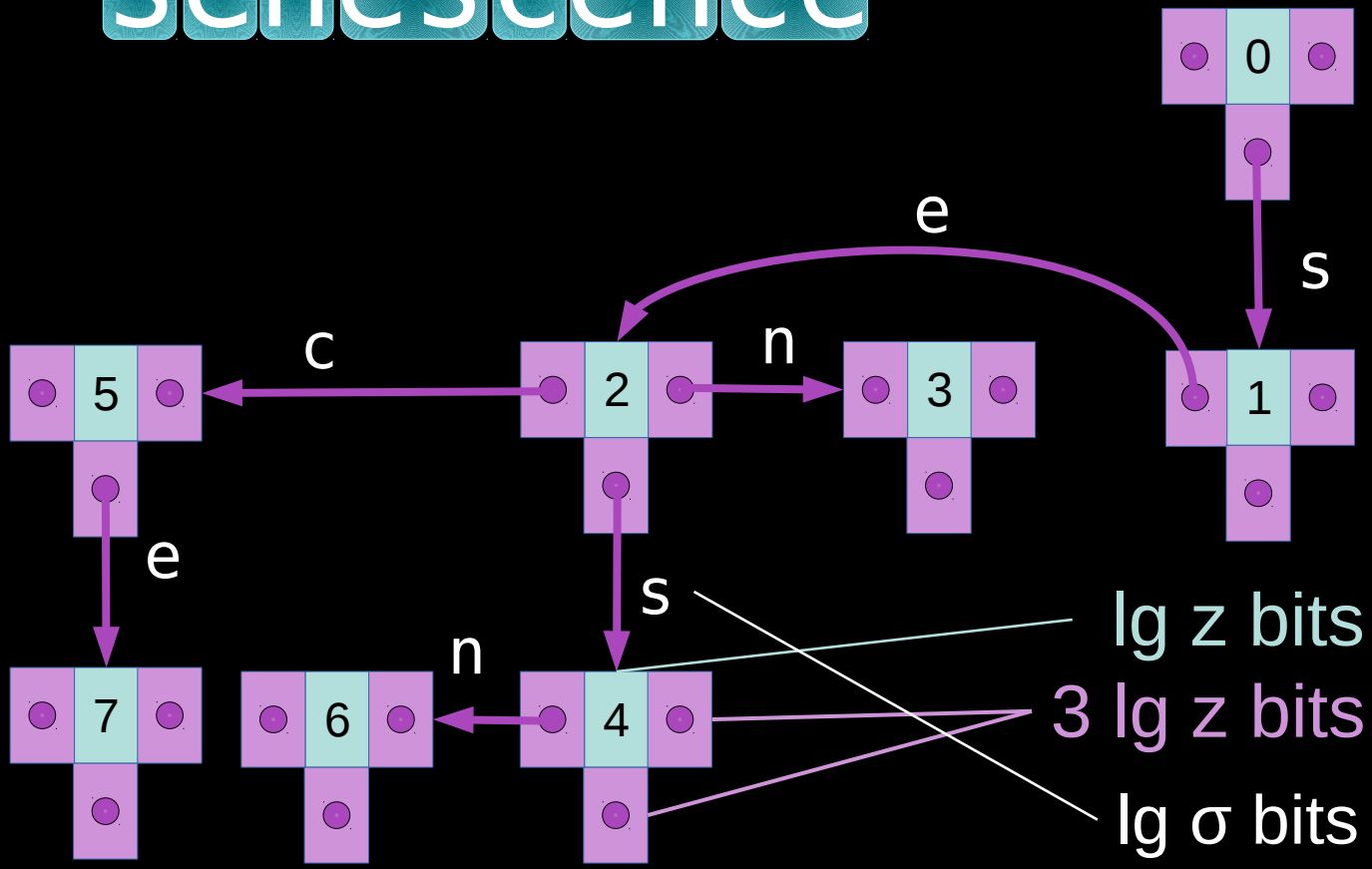
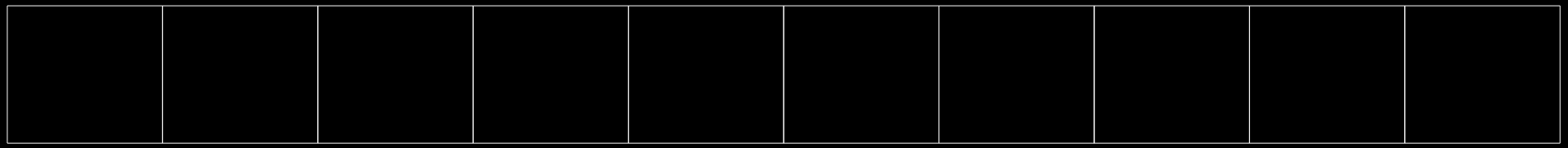


0

hash trie



senescence

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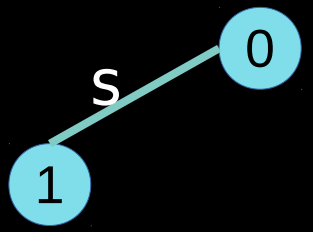
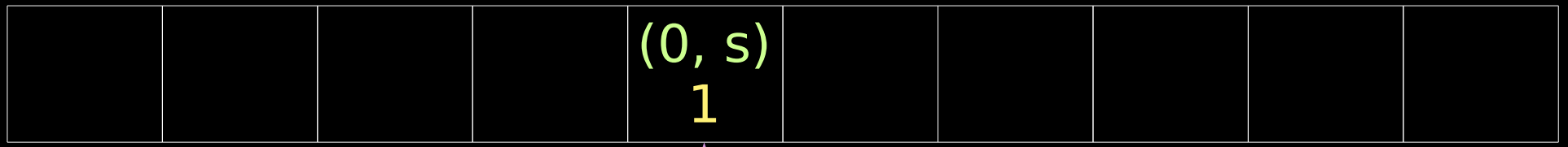


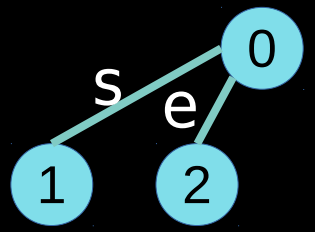


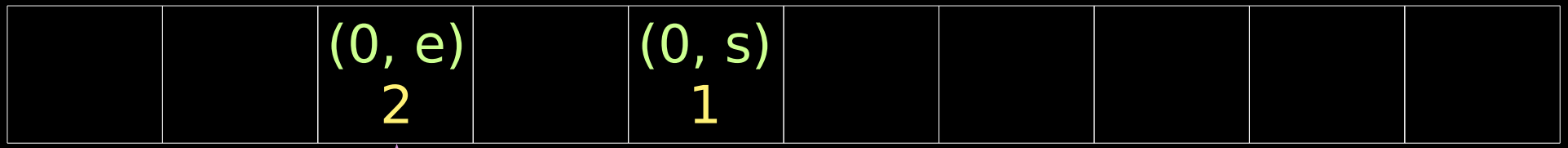
key (0, s)
value '1'

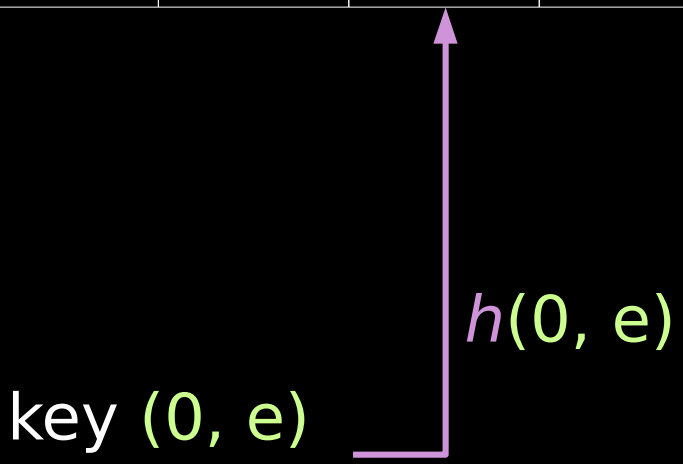
h(0, s)

senescence

hash trie







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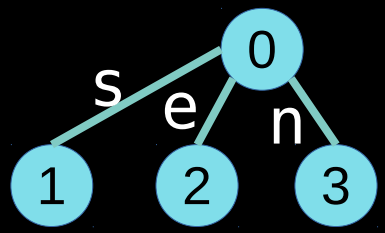


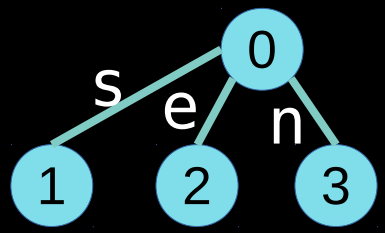
		(0, e) 2		(0, s) 1		(0, n) 3			

h(0, 2)

key (0, n)
value '3'

senescence

hash trie



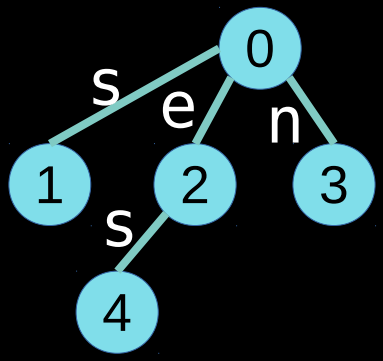
		(0, e) 2		(0, s) 1		(0, n) 3			

h(0, e)

key (0, e)
value '2'

senescence

hash trie



		(0, e) 2		(0, s) 1		(0, n) 3		(2, s) 4	

h(2, s)

key (2, s)
value '4'

senescence

key (i, s)
value j

lg z bits
lg σ bits
lg z bits

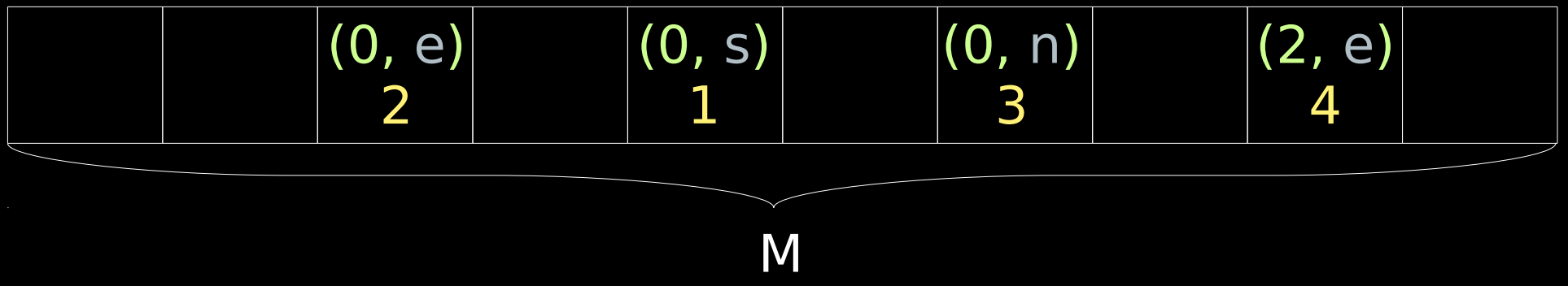
$2\lg z + \lg \sigma$ bits

		(0, e) 2		(0, s) 1		(0, n) 3		(2, e) 4	

key (i, s)
value j

lg z bits
lg σ bits
lg z bits

$2\lg z + \lg \sigma$ bits



- table size M
- load factor α
- $z \leq \alpha M$

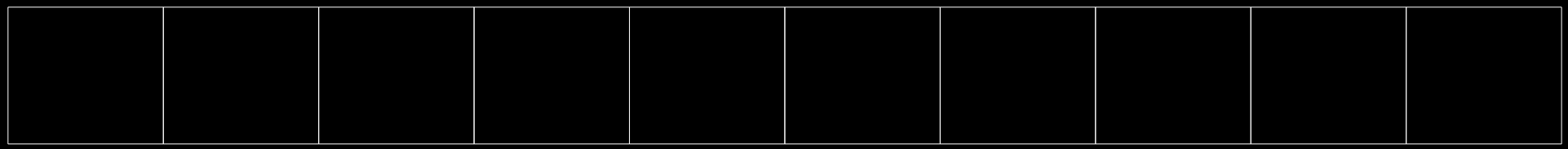
$$\geq \frac{z}{\alpha}(2\lg z + \lg \sigma) \text{ bits}$$

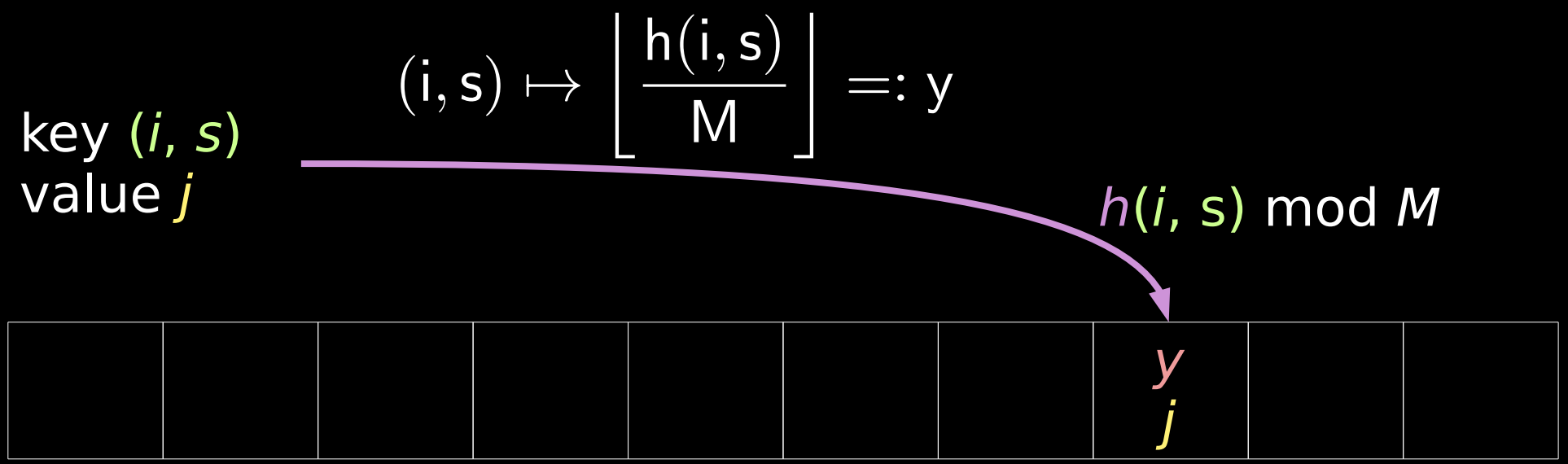
compact hash trie

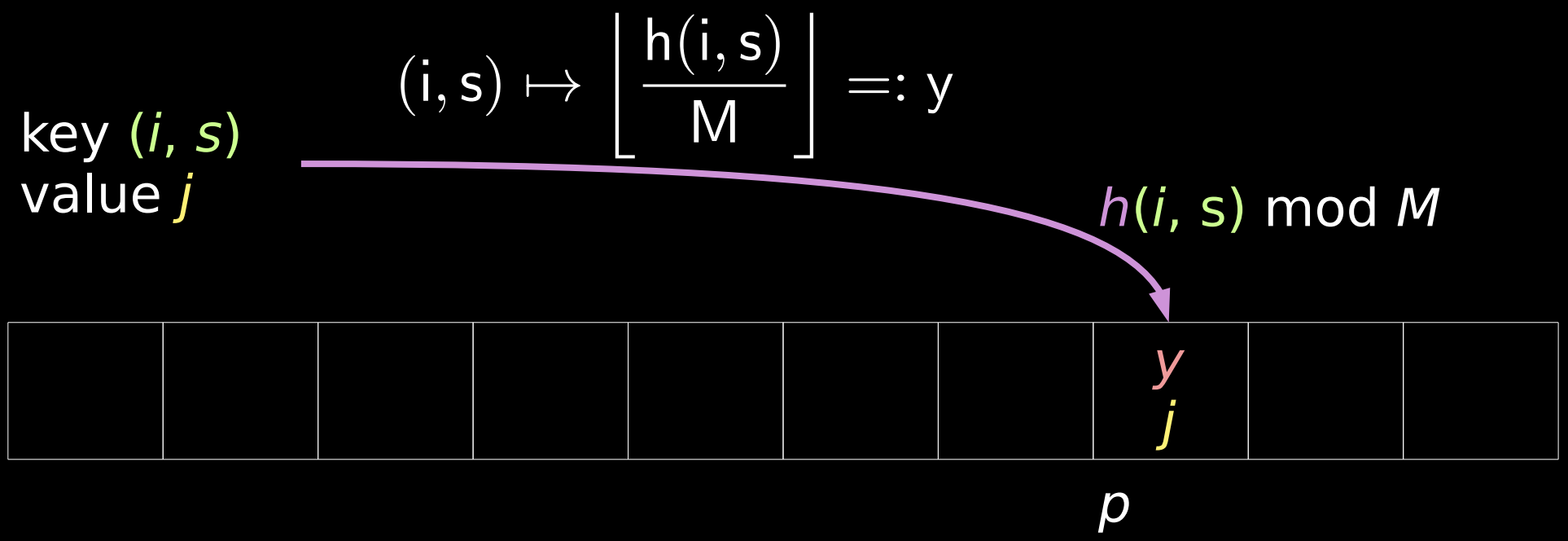
- bijective hash function
  - $(ax + b) \bmod q$
  - xorshift $x \longrightarrow x \oplus (x \gg j)$
- choose j, q so $h : [0..\sigma M] \longrightarrow [0..\sigma M]$ is bijective:
  - invariant: $z \leq \alpha M$
  - $| [0..z\text{-}1] \times [0..\sigma\text{-}1] | = z\sigma \leq \alpha\sigma M \leq \sigma M$

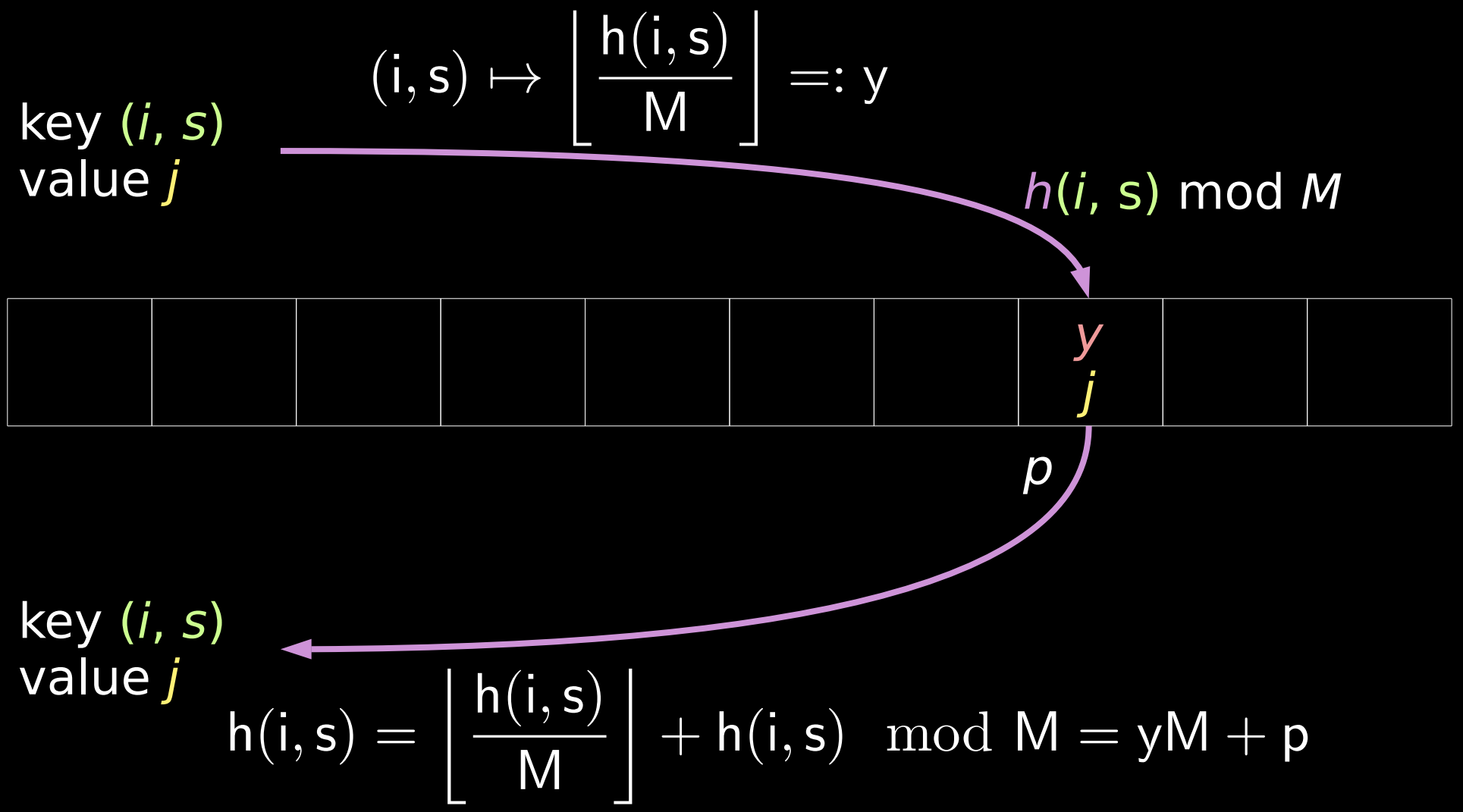
based on [Poyias and Raman'17]

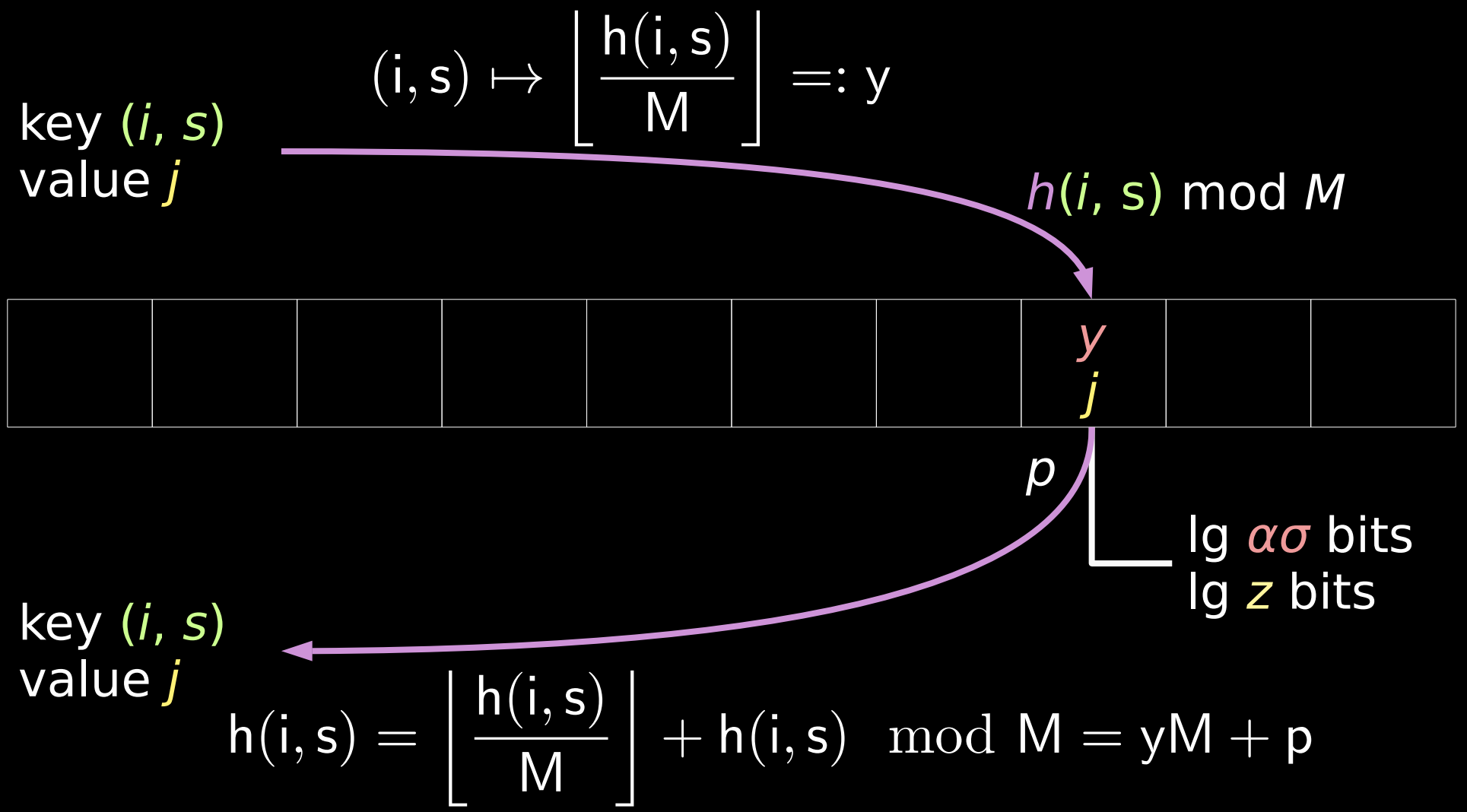
key (i, s)
value j




$(\mathrm{i}, \mathrm{s}) \mapsto \left\lfloor \frac{\mathrm{h}(\mathrm{i}, \mathrm{s})}{\mathrm{M}} \right\rfloor =: \mathrm{y}$
key (*i*, *s*)
value *j*
h(*i*, s) mod *M*
y
j


$(\mathrm{i}, \mathrm{s}) \mapsto \left\lfloor \frac{\mathrm{h}(\mathrm{i}, \mathrm{s})}{\mathrm{M}} \right\rfloor =: \mathrm{y}$
key (*i*, *s*)
value *j*
h(*i*, s) mod *M*
y
j
p

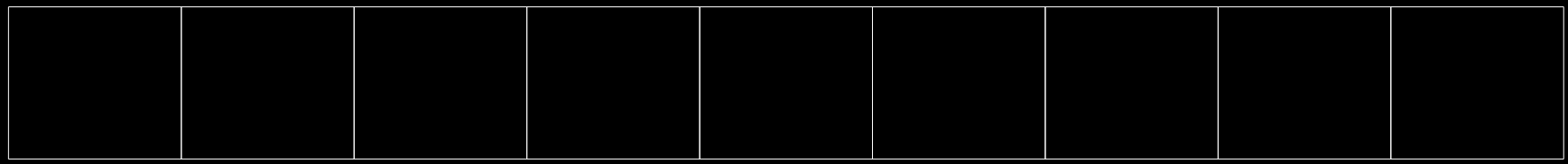

$(\mathsf{i},\mathsf{s}) \mapsto \left\lfloor \frac{\mathsf{h}(\mathsf{i},\mathsf{s})}{\mathsf{M}} \right\rfloor =: \mathsf{y}$
key (*i*, *s*)
value *j*
h(*i*, s) mod *M*
y
j
p
key (*i*, *s*)
value *j*
$\mathsf{h}(\mathsf{i},\mathsf{s}) = \left\lfloor \frac{\mathsf{h}(\mathsf{i},\mathsf{s})}{\mathsf{M}} \right\rfloor + \mathsf{h}(\mathsf{i},\mathsf{s}) \mod \mathsf{M} = \mathsf{yM} + \mathsf{p}$


$(\mathrm{i},\mathrm{s}) \mapsto \left\lfloor \frac{\mathrm{h}(\mathrm{i},\mathrm{s})}{\mathrm{M}} \right\rfloor =: \mathrm{y}$
key (*i*, *s*)
value *j*
h(*i*, s) mod *M*
y
j
p
lg *ασ* bits
lg *z* bits
key (*i*, *s*)
value *j*
$\mathrm{h}(\mathrm{i},\mathrm{s}) = \left\lfloor \frac{\mathrm{h}(\mathrm{i},\mathrm{s})}{\mathrm{M}} \right\rfloor + \mathrm{h}(\mathrm{i},\mathrm{s}) \mod \mathrm{M} = \mathrm{yM} + \mathrm{p}$

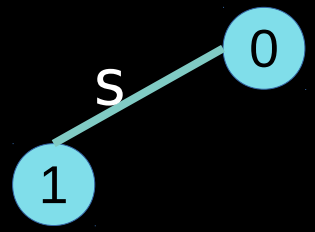
0

rolling hash trie

senescence

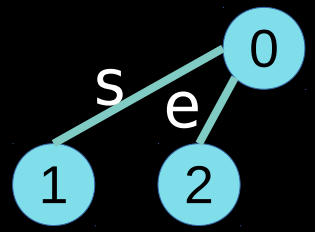


rolling hash trie

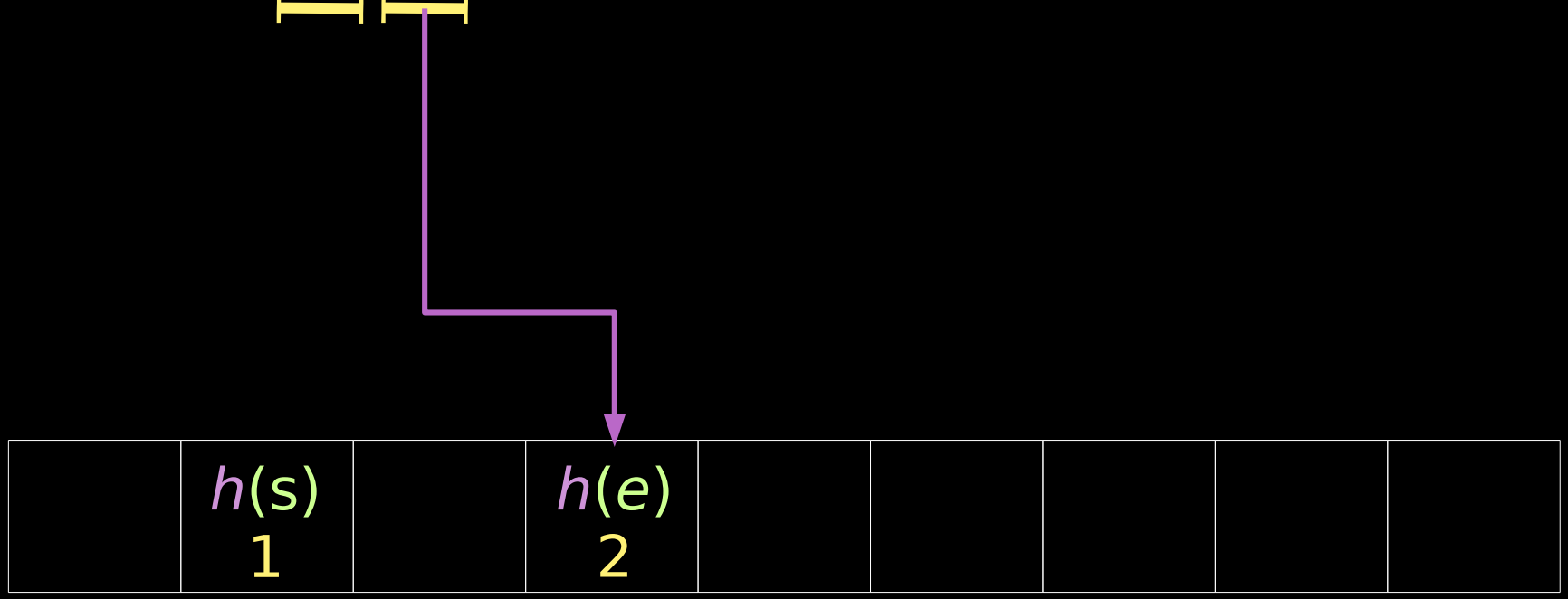


senescence

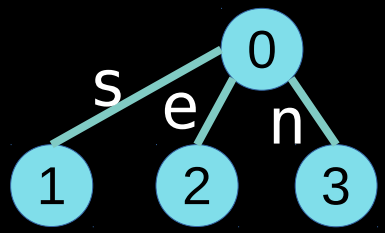
rolling hash trie



senescence



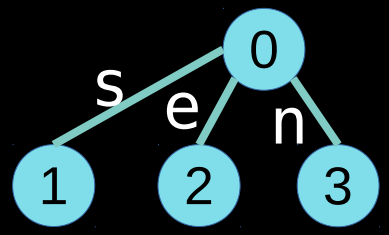
rolling hash trie



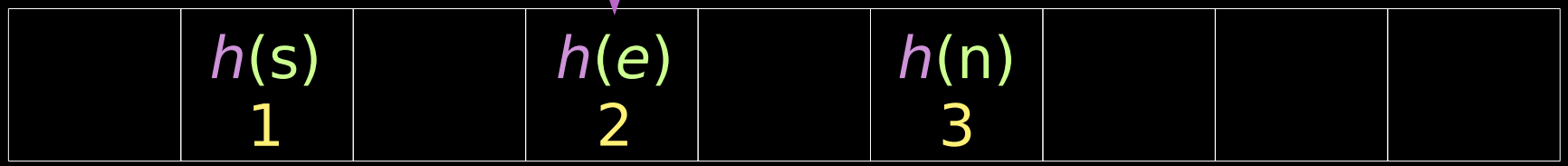
senescence

	h(s) 1		*h*(*e*) 2		*h*(n) 3			

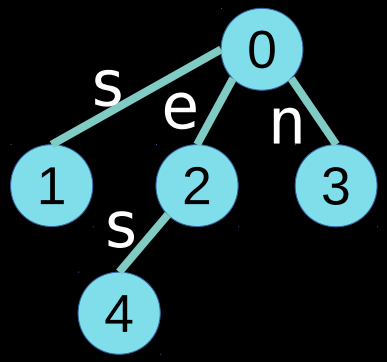
rolling hash trie



senescence



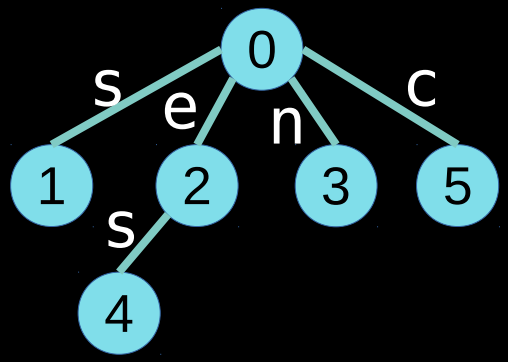
rolling hash trie



senescence

	h(s) 1		*h*(*e*) 2		*h*(n) 3		*h*(es) 4	

rolling hash trie



senescence

h(c) = *h*(es)

	h(s) 1		*h*(e) 2		*h*(n) 3		*h*(es) 4	

properties

- Monte Carlo algorithm
- store fingerprint instead of keys

	h(s) 1		*h*(*e*) 2		*h*(n) 3		*h*(es) 4	

w: integer width (64/128 bit integer)

properties

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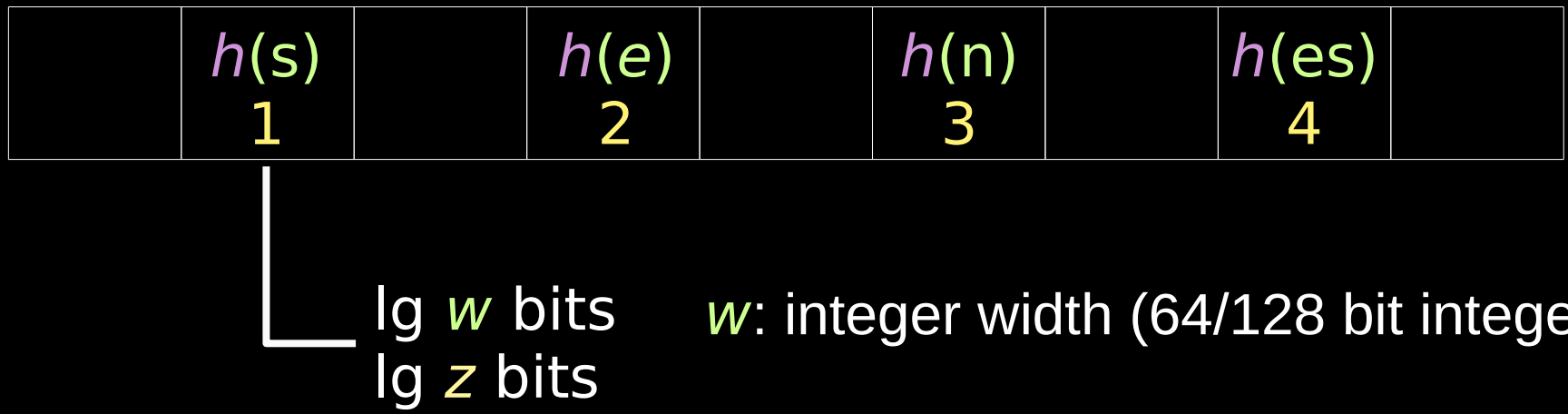
- store fingerprint instead of keys

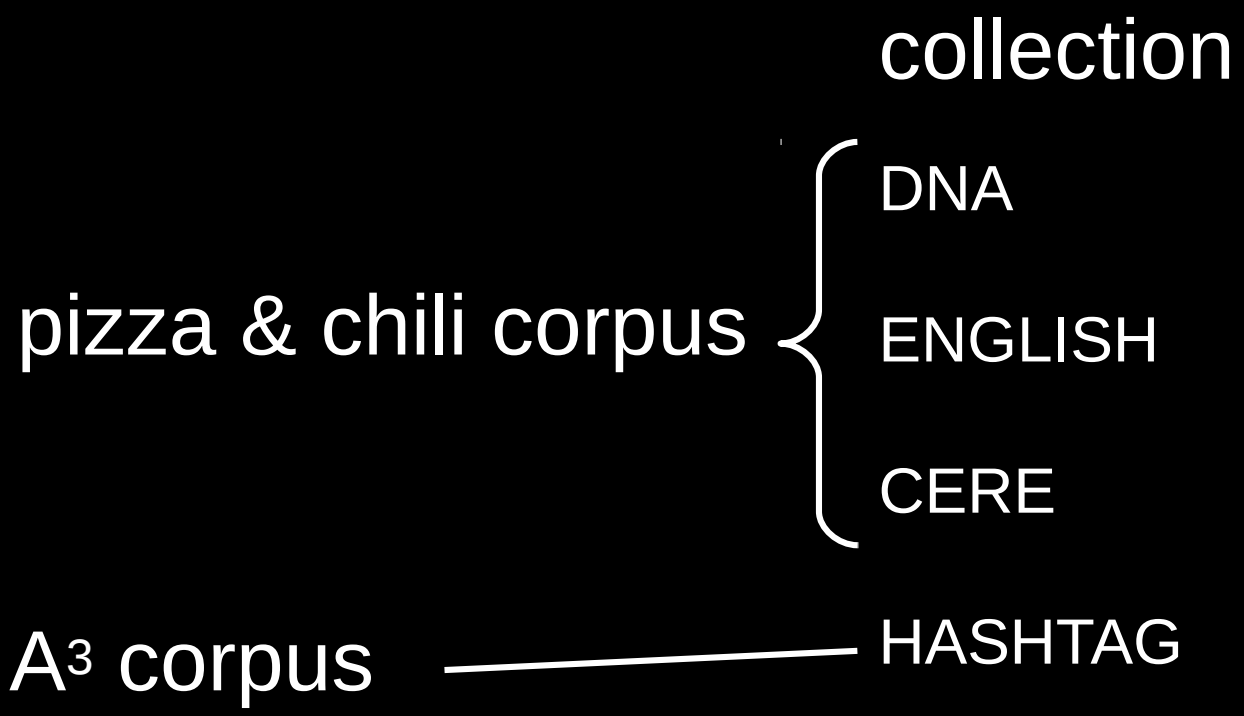


space

data structure	bits
binary trie	$z\,(2 \lg z + \lg \sigma)$
ternary trie	$z\,(3 \lg z + \lg \sigma)$
hash trie	$z/\alpha\,(2 \lg z + \lg \sigma)$
compact hash trie	$z/\alpha \lg \alpha z\sigma$
rolling hash trie	$z/\alpha \lg wz$

datasets

200 MiB text files



technical details

type sizes

- byte alphabet ($\sigma = 256$)
- $w = 64$ bits

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hashing

- linear probing
- hash functions:
  - splitmix64 (Vigna'15)
  - rolling:

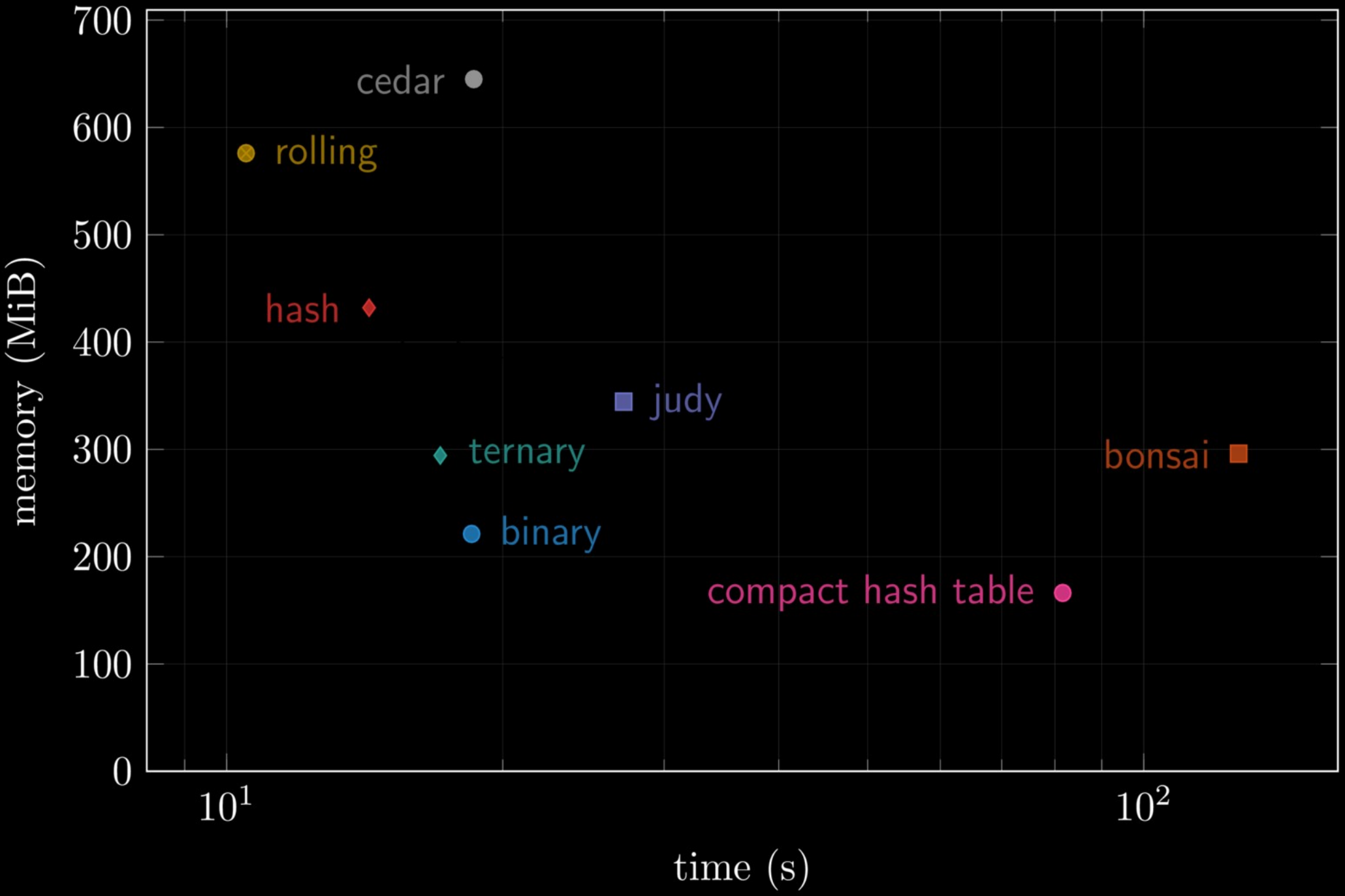
$$h(T) := \sum_{i=1}^{|T|} T[i](\sigma+1)^{|T|-i} \mod 2^{64}$$

σ+1 = 257 prime number

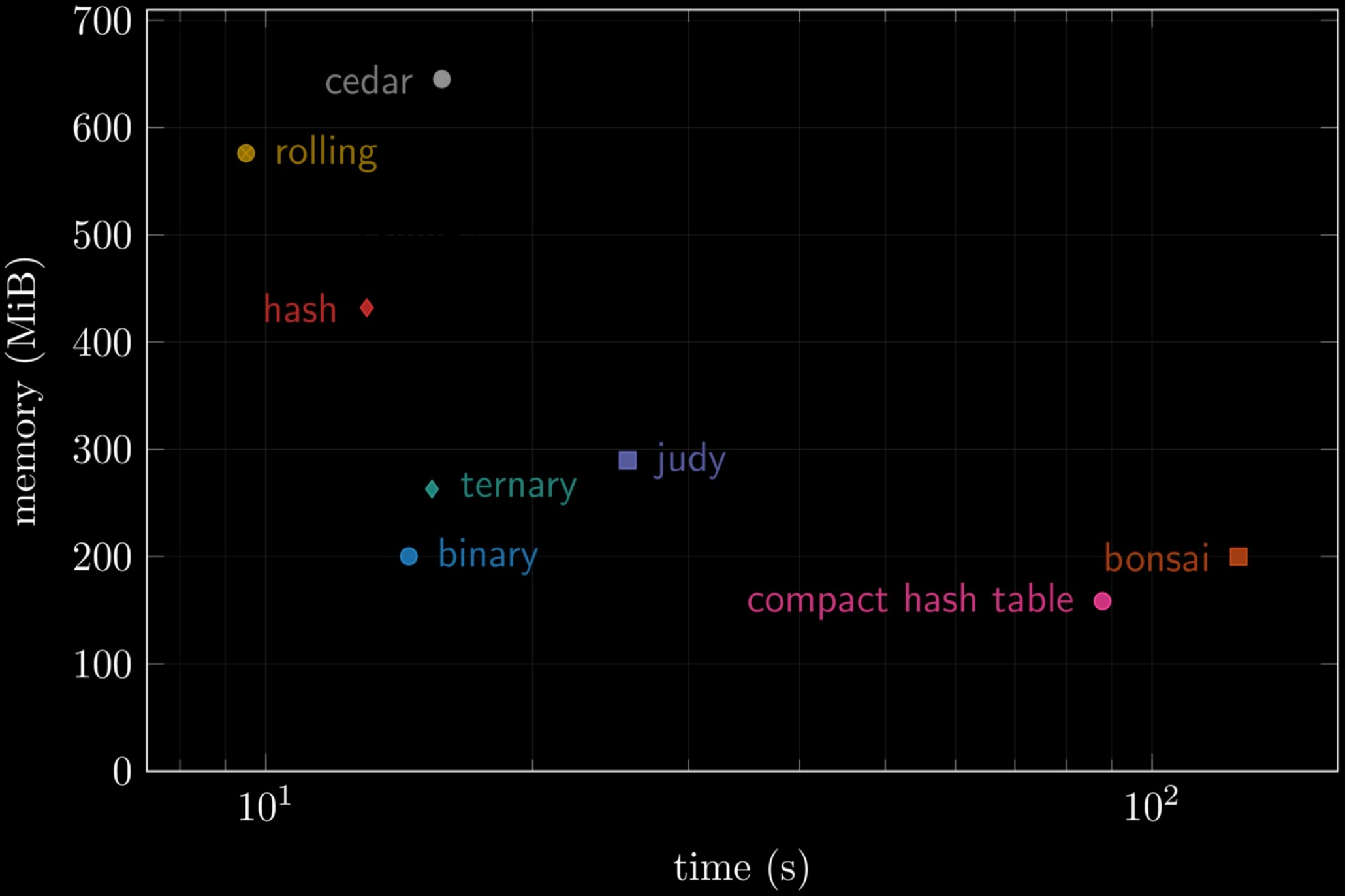
competitors

- judy array [Baskins'02]
  - 256-ary radix tries
- cedar [Yoshinaga and Kitsuregawa'14]
  - double array variant
- bonsai [Poyias and Raman'15]
  - trie with compact hashing

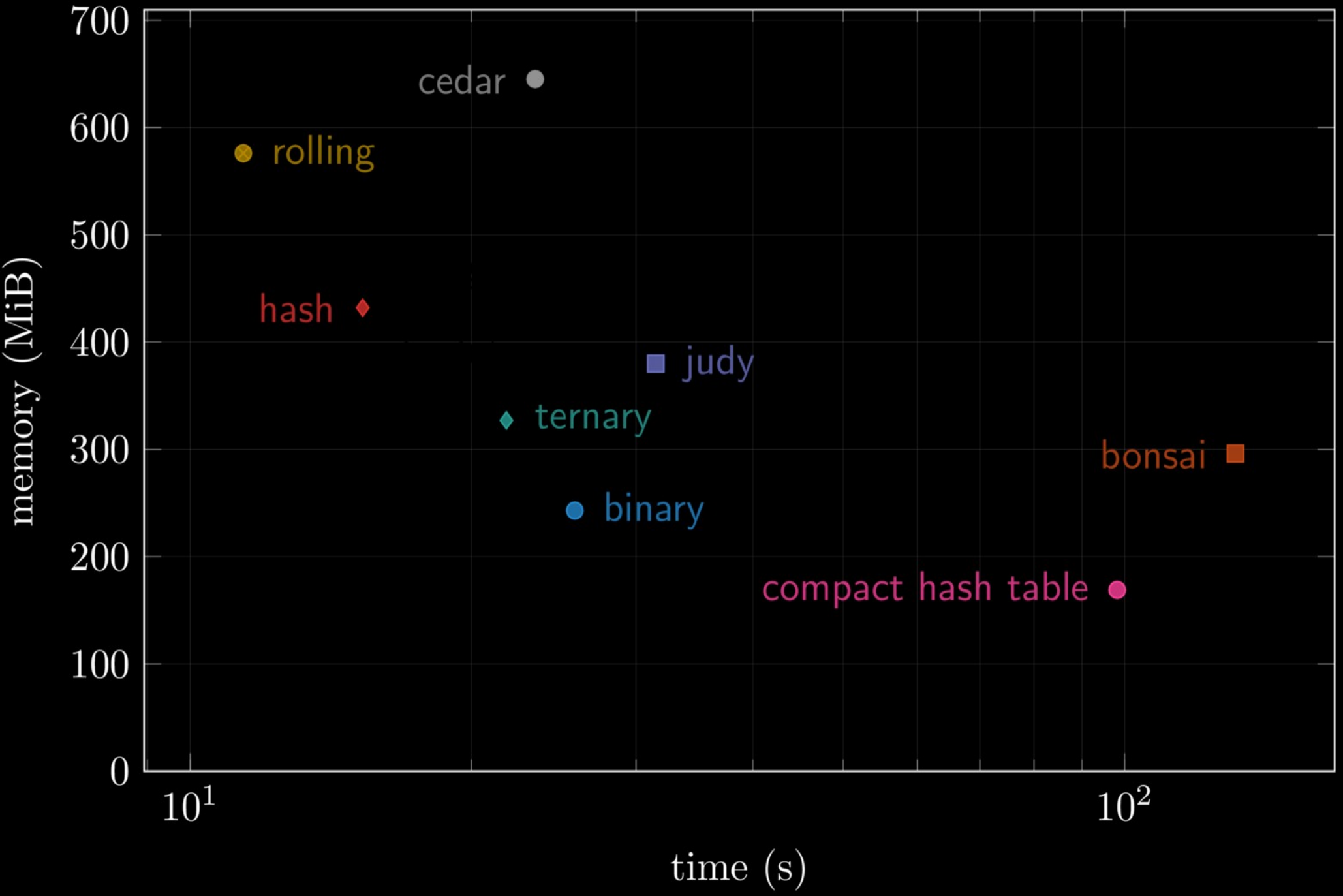
HASHTAG


700
600
500
400
300
200
100
0
memory (MiB)
cedar
rolling
hash
judy
ternary
bonsai
binary
compact hash table
10^1
10^2
time (s)

PC-DNA


700
600
500
400
300
200
100
0
memory (MiB)
cedar
rolling
hash
judy
ternary
binary
bonsai
compact hash table
10^1
10^2
time (s)

PC-ENGLISH


700
600
500
400
300
200
100
0
memory (MiB)
cedar
rolling
hash
judy
ternary
bonsai
binary
compact hash table
10^1
10^2
time (s)

PCR-CERE


600
500
400
300
200
100
0
memory (MiB)
rolling
hash
cedar
judy
ternary
binary
bonsai
compact hash table
10^1
10^2
time (s)

conclusion

- rolling fastest, but output maybe corrupt
- hash fast + correct
- ternary best trade-off speed ↔ memory
- binary slower, but less memory
- compact hash trie memory friendliest
- competitors not in Pareto front

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Thank you for your attention. Any questions are welcome!