Computing All Distinct Squares in Linear Time for Integer Alphabets

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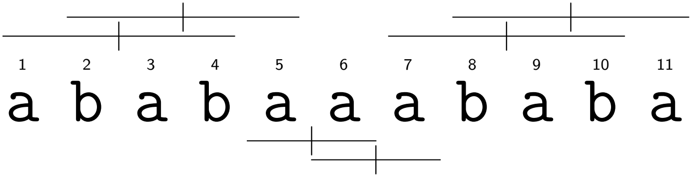
with python a one-liner: `[i**2 for i in range(1,n)]`

squares

1	2	3	4	5	6	7	8	9	10	11
a	b	a	b	a	a	a	b	a	b	a

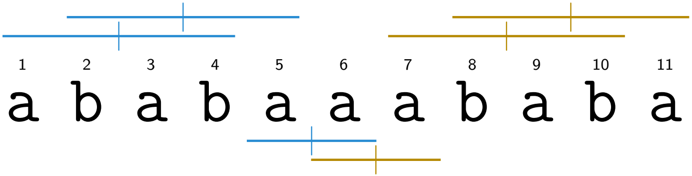
squares



squares

- abab at 1
- baba at 2
- aa at 5
- aa at 6
- abab at 7
- baba at 8

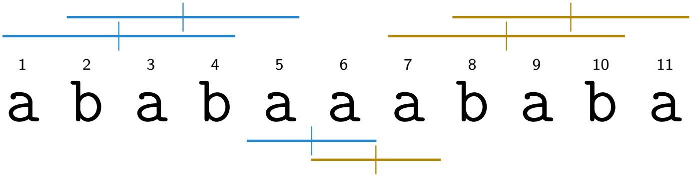
squares



leftmost squares

- abab at 1
- baba at 2
- aa at 5
- ~~aa at 6~~
- ~~abab at 7~~
- ~~baba at 8~~

squares



leftmost squares

- abab at 1
- baba at 2
- aa at 5
- ~~aa at 6~~
- ~~abab at 7~~
- ~~baba at 8~~

fact

leftmost squares $\equiv$ distinct squares

squares = tandem repeats

GATCAATGAGGTGGTACATGCTAGTACACGCGAACACGCGA

tandem repeats

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- AA

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why?

- genetic fingerprint
- unterstand DNA better
- combinatorial (interesting)
- for compression?

setting

given

- text T
- $n := |T|$ text length
- alphabet of size $n^{\mathcal{O}(1)}$

problem

find all distinct squares

goal: $\mathcal{O}(n)$ time

naive solution

- iterate over each text position i
- iterate over all possible periods p
- compare $T[i+c] \stackrel{!}{=} T[i+p+c]\ \forall c = 0, \ldots, p-1$
- if found a square $\Rightarrow$ check whether already reported

$$\Rightarrow \mathcal{O}\left(\underbrace{n}_{\forall i} \cdot \underbrace{n}_{\forall p} \cdot \underbrace{n}_{\forall c} \cdot t_\lambda\right)$$

- t_λ: time for look-up ($t_\lambda = n \lg \sigma$ for a simple trie)

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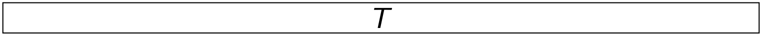
- use LCP data structure to check characters in $\mathcal{O}(1)$ time

LCP data structure

- given string T
- construction in linear time
- answers in $\mathcal{O}(1)$ time



LCP data structure

- given string T
- construction in linear time
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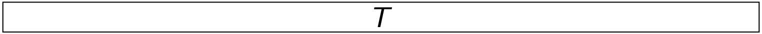

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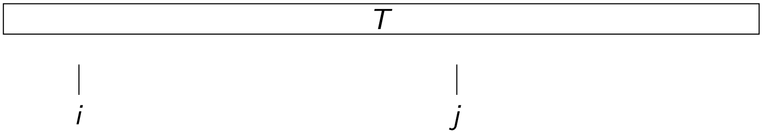

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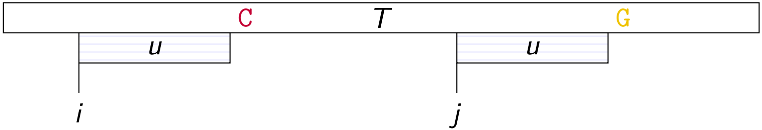

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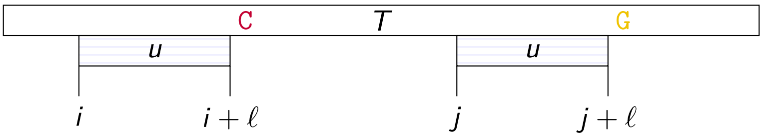

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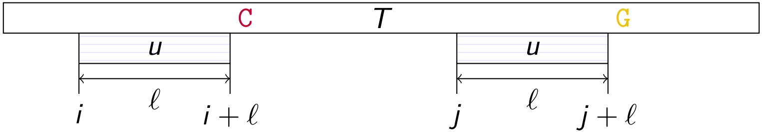

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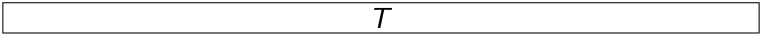

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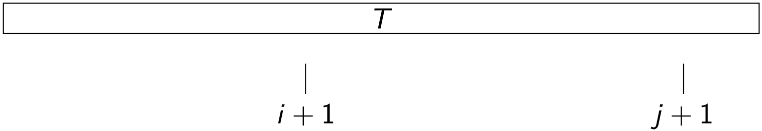

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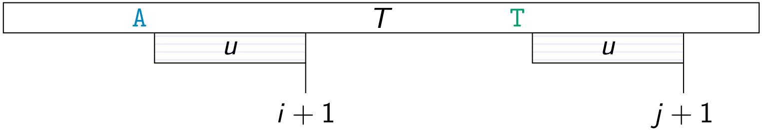

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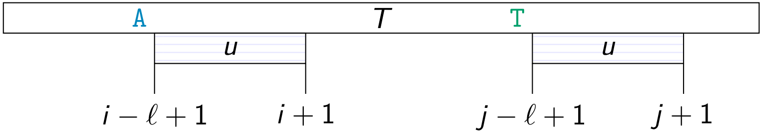

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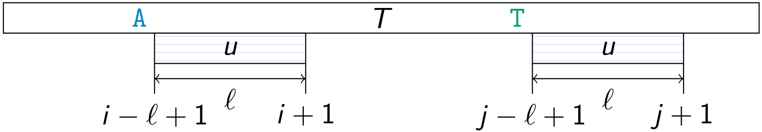

better solutions

idea to get faster

- check only at certain text position
- check only periods up to a threshold

sufficient: all borders of Lempel-Ziv factors

idea from

[Gusfield and Stoye'04]

computing all distinct squares in $\mathcal{O}(\sigma n)$ time

Lempel-Ziv parsing

a b a b a a a b a b a

Lempel-Ziv parsing

a | b a b a a a b a b a

Lempel-Ziv parsing

a | b | a b a a a b a b a

Lempel-Ziv parsing

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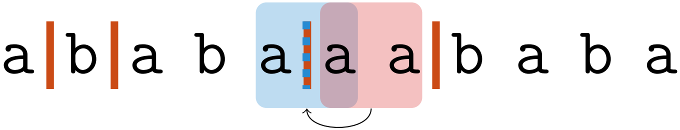
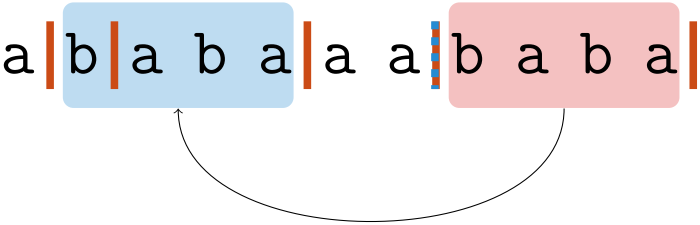


Lempel-Ziv parsing



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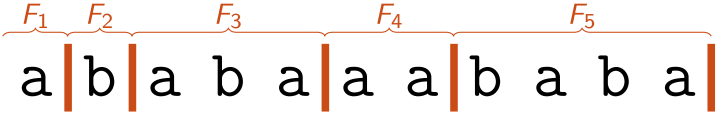
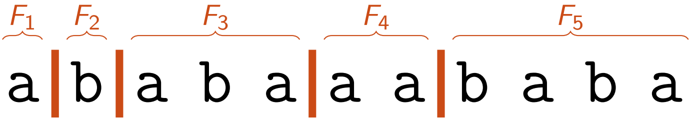


computing squares

a b a b a a a b a b a

reported squares:

computing squares



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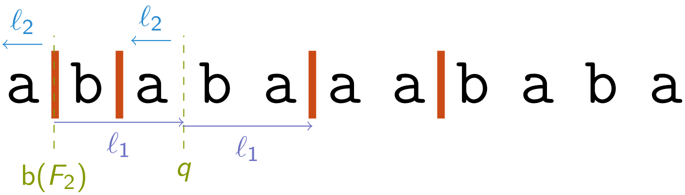

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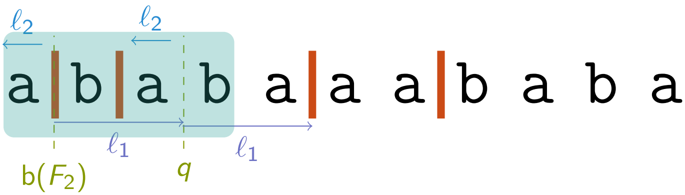
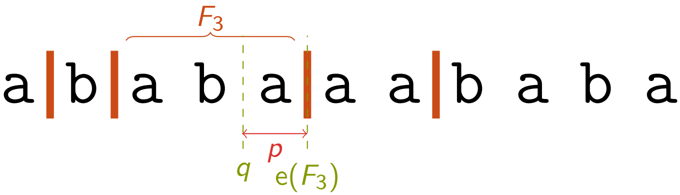


reported squares:

- abab

computing squares



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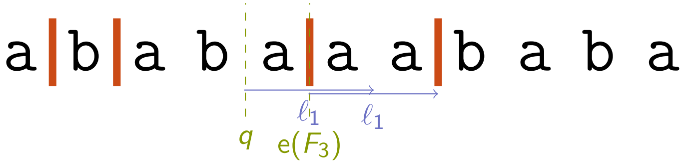

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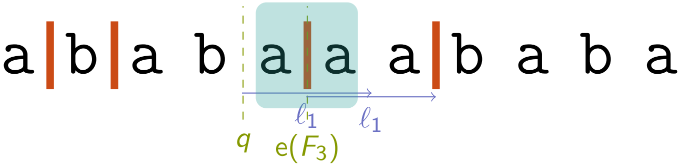

reported squares:

- abab
- aa

computing squares

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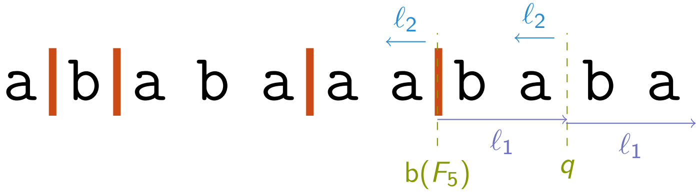

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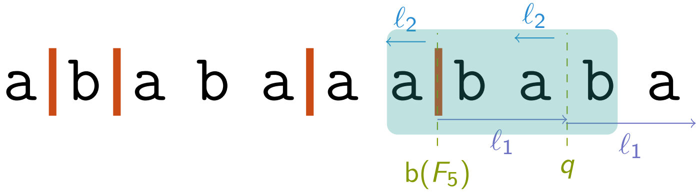


reported squares:

- abab
- aa
- abab

computing squares

a | b | a b a | a a | b a b a

reported squares:

- abab
- aa
- abab

problems:

- reporting duplicates
- baba not found

$\mathcal{O}(n)$ time goal

problem

- $\nexists$ dictionary with $\mathcal{O}(1)$ access/update time
- store lists, be careful about uniqueness!

solution

use LPF array!

longest previous factor table

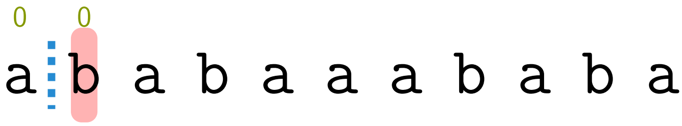
a b a b a a a b a b a

longest previous factor table

0

a b a b a a a b a b a

longest previous factor table



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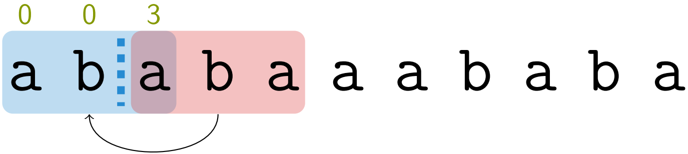

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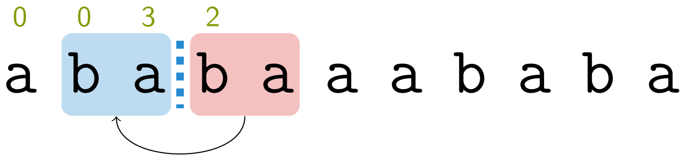

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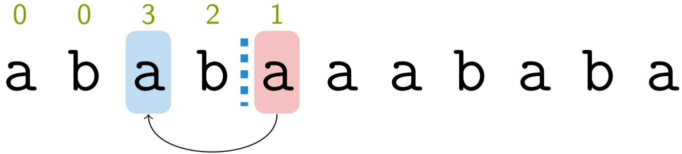

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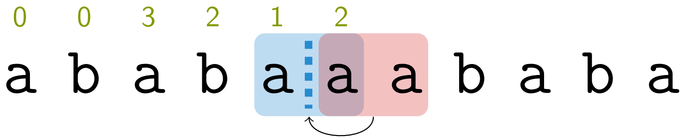

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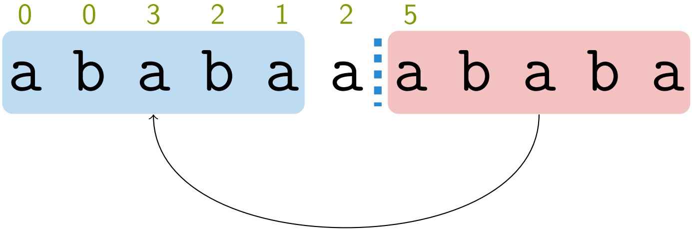

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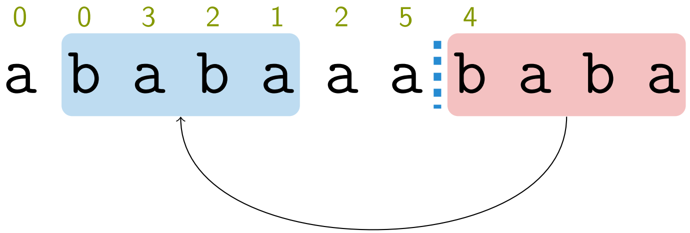

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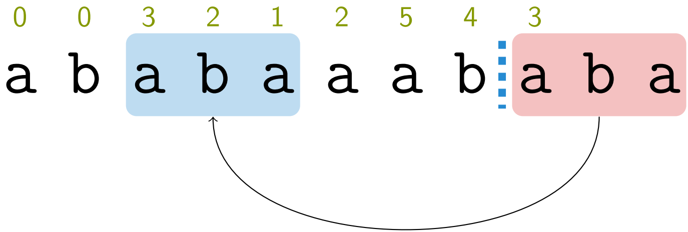

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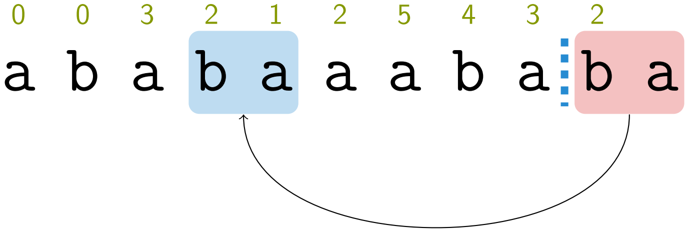

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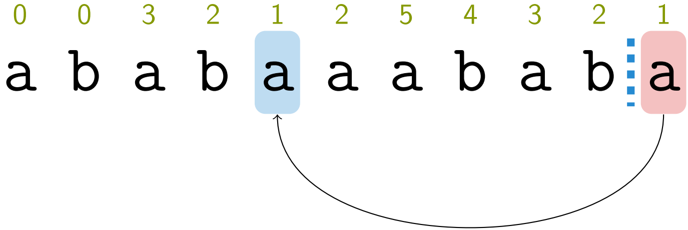
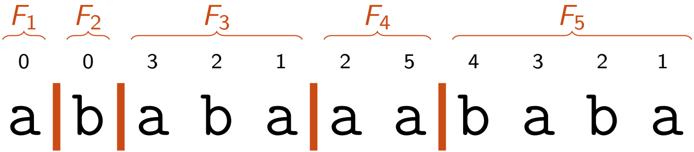


computing squares

0	0	3	2	1	2	5	4	3	2	1
a	b	a	b	a	a	a	b	a	b	a

reported squares:

computing squares



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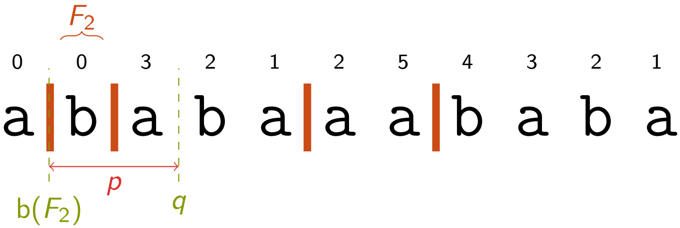

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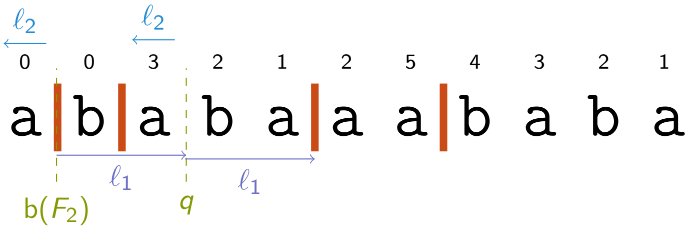

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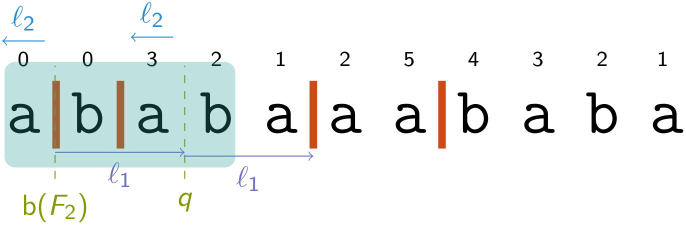

reported squares:

- abab

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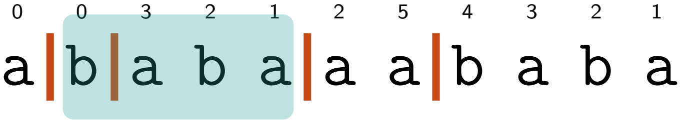


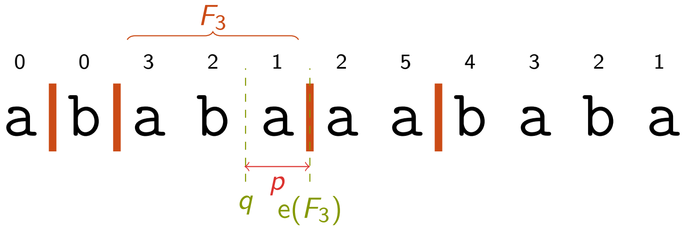
reported squares:

- abab
- baba

new techniques:

- right rotate found squares

computing squares



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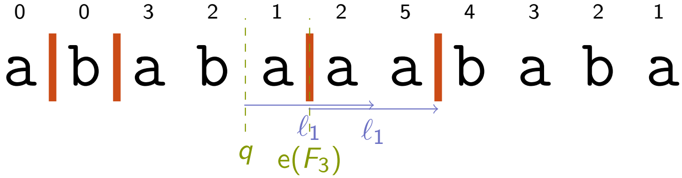

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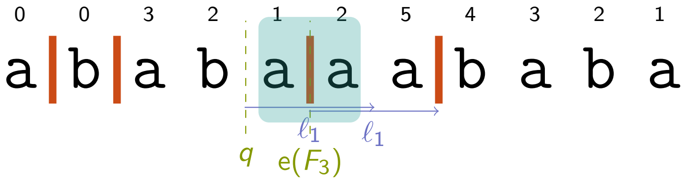

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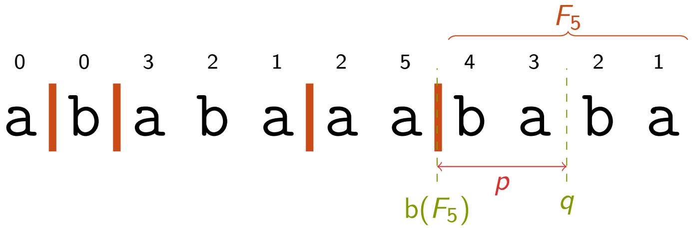

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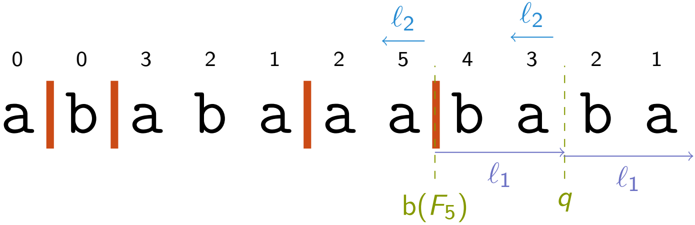

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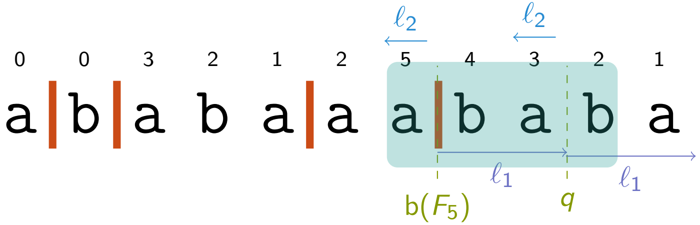

reported squares:

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- aa

new techniques:

- right rotate found squares
- skip if LPF[i] $> 2p$

experiments

collection	σ	z	$\max_x \lvert F_x \rvert$	$\lvert occ \rvert$	time
`dblp.xml`	97	7,035,342	1060	7412	70
`proteins`	26	20,875,097	45,703	3,108,339	245
`dna`	17	13,970,040	97,966	132,594	310
`english`	226	13,971,134	987,766	13,408	2639
`einstein`	125	49,575	906,995	18,192,737	3953

- 200 MiB collections from Pizza&Chili corpus
- σ: alphabet size
- z: # Lempel-Ziv factors
- time in seconds

summary

finding all distinct squares in $\mathcal{O}(n)$ time

techniques

- modification of [Gusfield and Stoye'04]
- using LPF array

further linear time results (read the paper!)

- decorating suffix tree with information of all squares
- building topology of the minimal augmented suffix tree (MAST)

open problems

- create MAST in $\mathcal{O}(n)$ time

summary

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- using LPF array

further linear time results (read the paper!)

- decorating suffix tree with information of all squares
- building topology of the minimal augmented suffix tree (MAST)

open problems

- create MAST in $\mathcal{O}(n)$ time

Thank you for listening. Any questions are welcome!