Indexing the Bijective BWT

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FM Index

ingredients

- BWT
- wavelet tree

FM Index

ingredients

- BWT
- wavelet tree

operation: backward search

- locate pattern
- #occ: number of occurrences
- time independent on #occ

FM Index on bijective BWT

ingredients

- bijective BWT
- wavelet tree

operation: backward search

- locate pattern
- #occ: number of occurrences
- time independent on #occ

FM Index

on bijective BWT

ingredients

- bijective BWT
- wavelet tree

operation: backward search

- locate pattern
- #occ: number of occurrences
- time independent on #occ

reason:
bijective BWT is cool

Lyndon words

- a
- aabab

Lyndon word is smaller than
- any proper suffix
- any rotation

Lyndon words

- a
- aabab

Lyndon word is smaller than
- any proper suffix
- any rotation

not Lyndon words:

- abaab (rotation aabab smaller)
- abab (abab not smaller than suffix ab)

Lyndon factorization

- input: text T
- output: factorization $T_1 ... T_t$ with
  - T_i is Lyndon word
  - $T_i \geq_{\text{lex}} T_{i+1}$
  - factorization uniquely defined
  - linear time [Duval'88]

(Chen-Fox-Lyndon theorem)

properties [Duval' 88]

- T_t:
  - smallest Lyndon word
  - smallest suffix of T
- T_i primitive
- T_1 longest Lyndon prefix of T[1..]
- T_{i+1} longest Lyndon prefix of $T[|T_1 \cdots T_i|+1..]$

$\prec_\omega$

- $u \prec_\omega w :\Longleftrightarrow uuuu\ldots <_{\text{lex}} wwww\ldots$
- ab < aba but aba $\prec_\omega$ ab

abababab⋯
ababaaba⋯

bijective BWT of senescence

s|enes|cen|ce

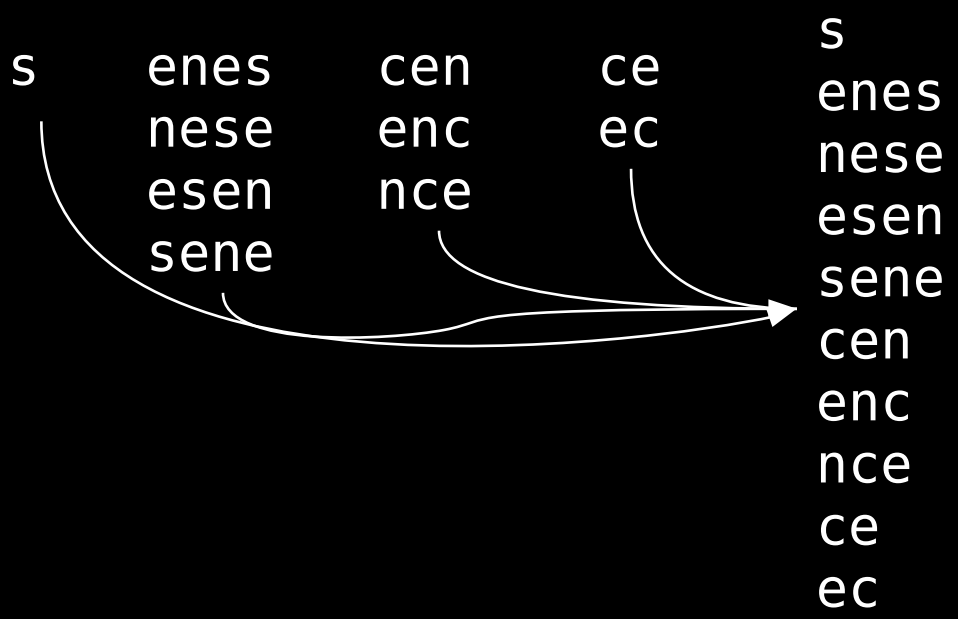
bijective BWT of senescence

s|enes|cen|ce

s	enes	cen	ce
	nese	enc	ec
	esen	nce	
	sene		

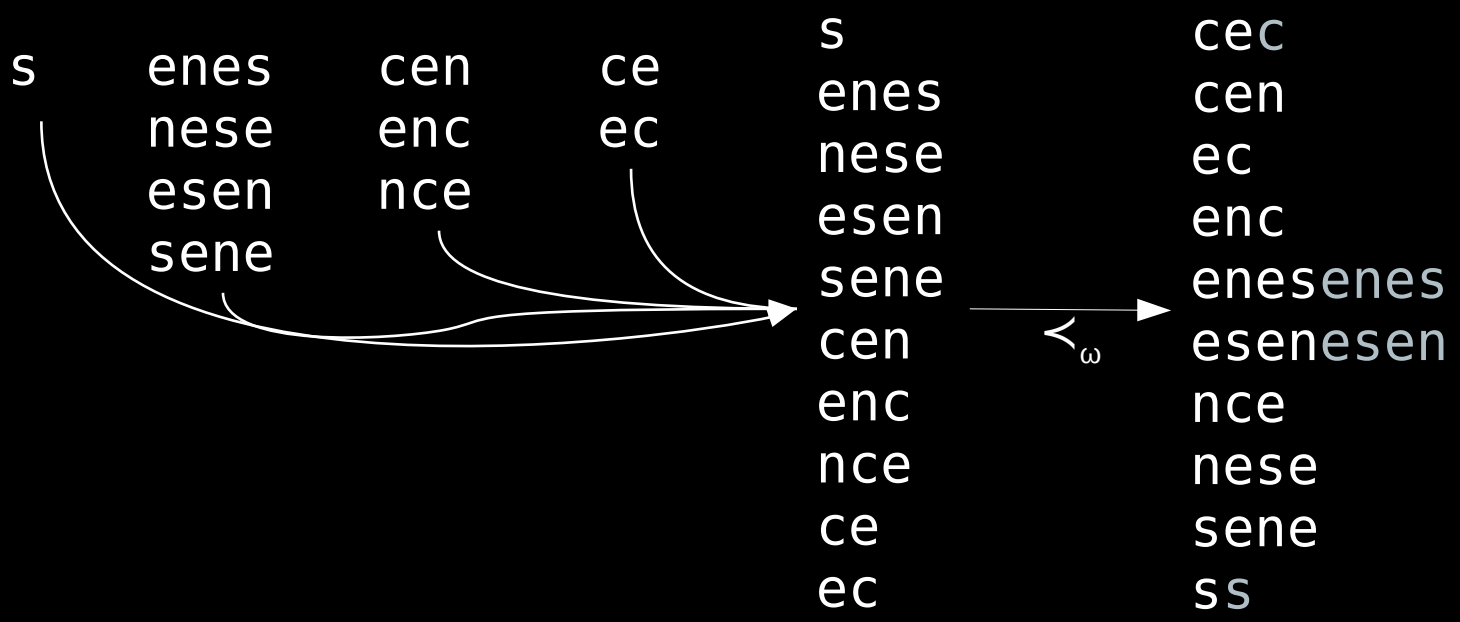
bijective BWT of senescence

s|enes|cen|ce



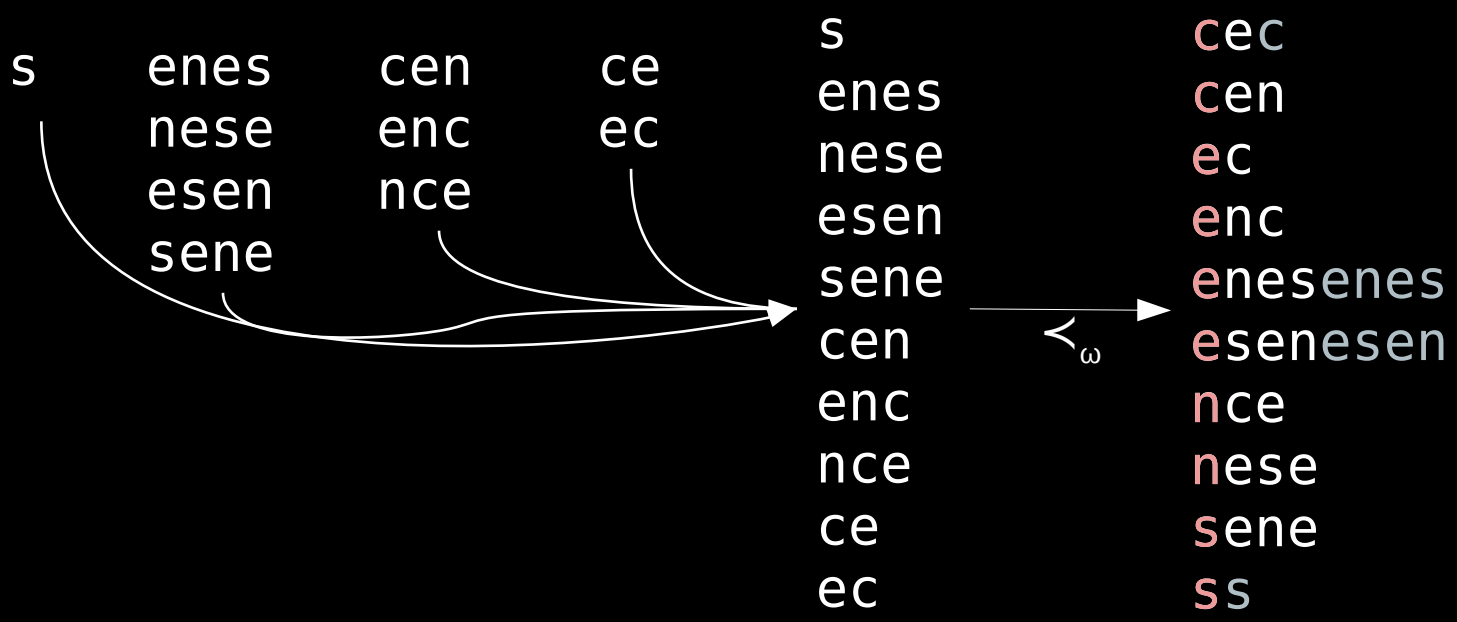
bijective BWT of senescence

s|enes|cen|ce



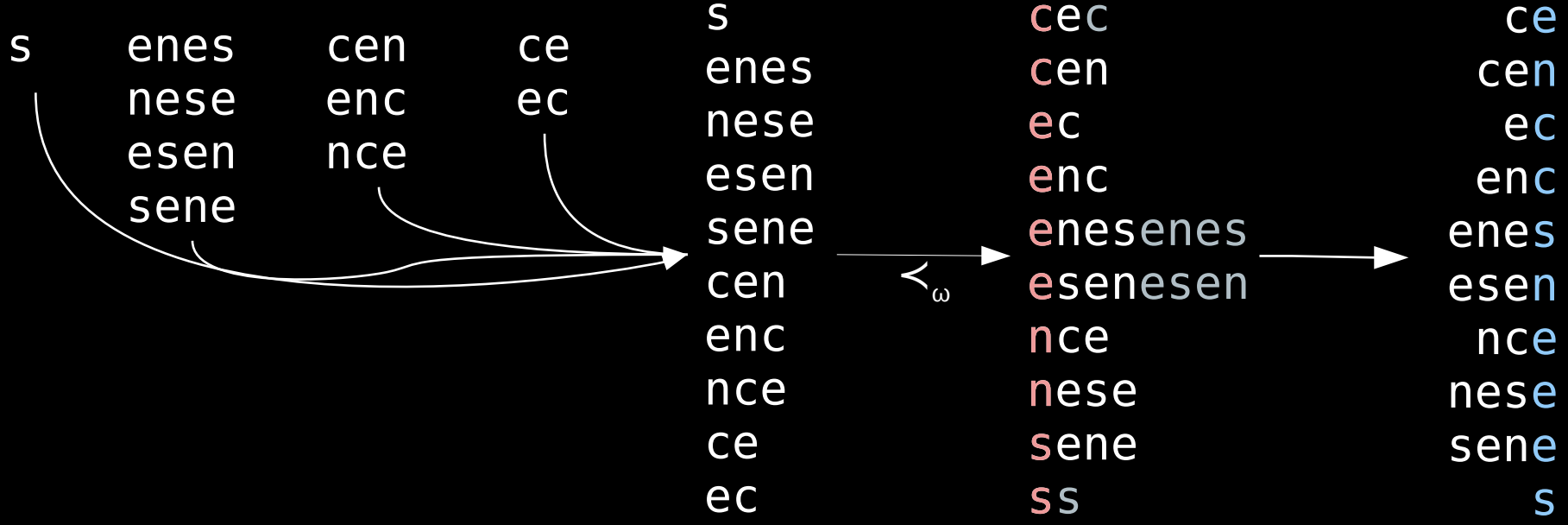
bijective BWT of senescence

s|enes|cen|ce



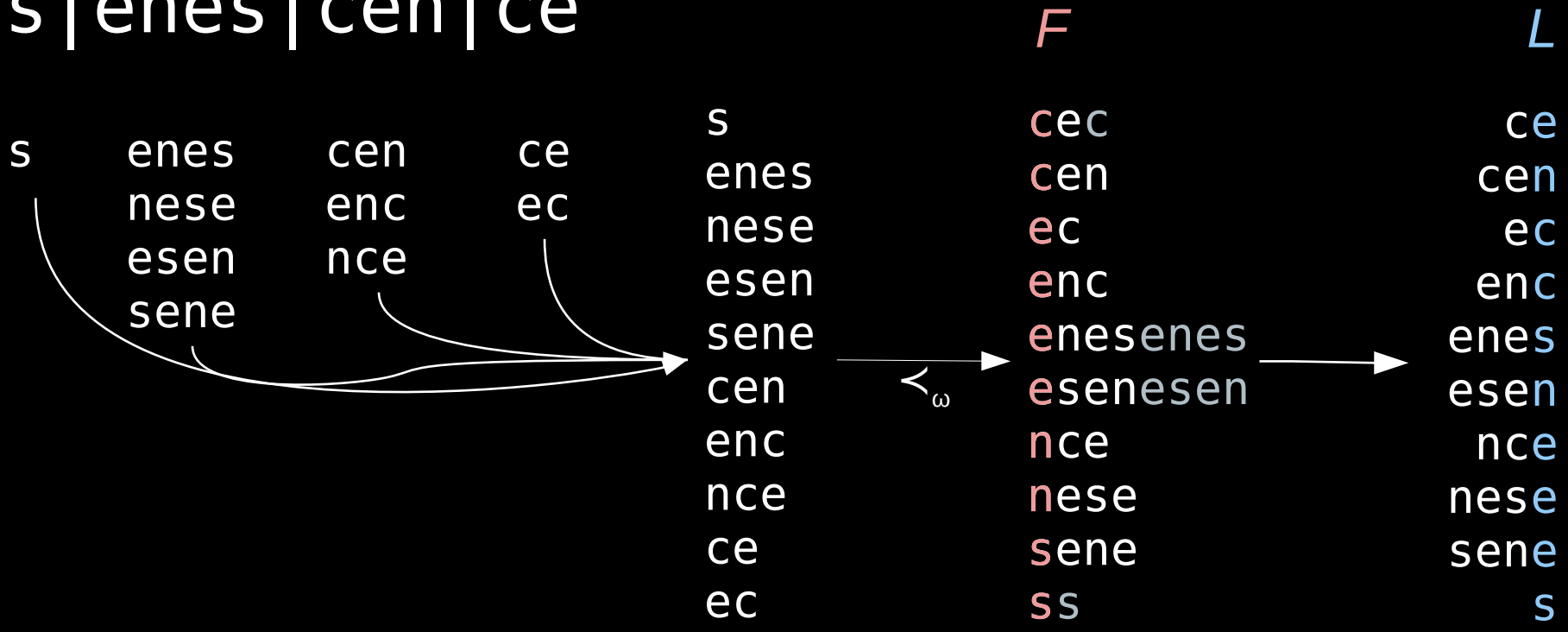
bijective BWT of senescence

s|enes|cen|ce



bijective BWT of senescence

s|enes|cen|ce

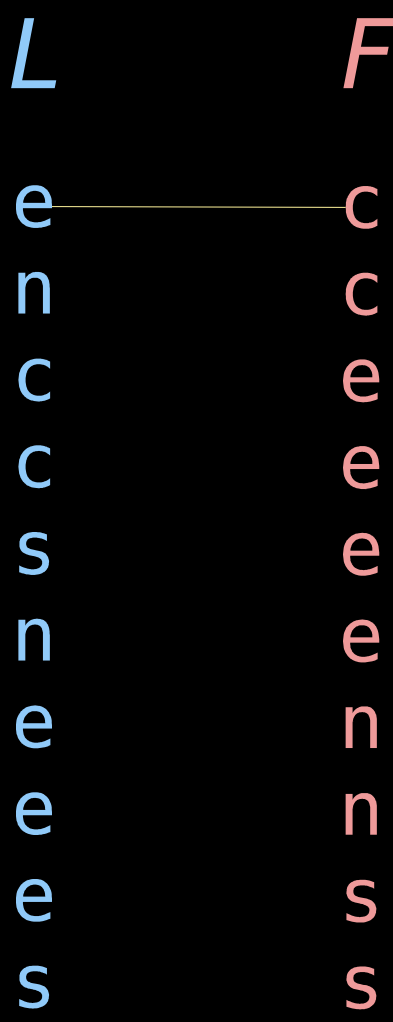


result: enccsneees

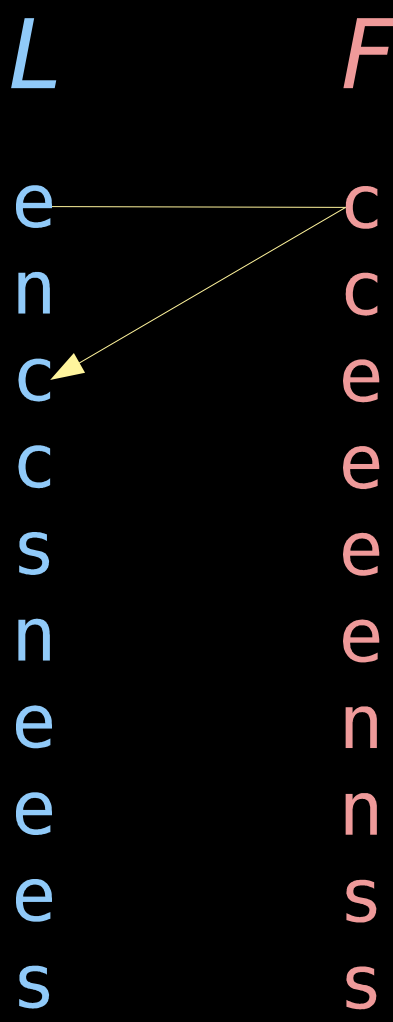
cycles

L	*F*
e	c
n	c
c	e
c	e
s	e
n	e
e	n
e	n
e	s
s	s

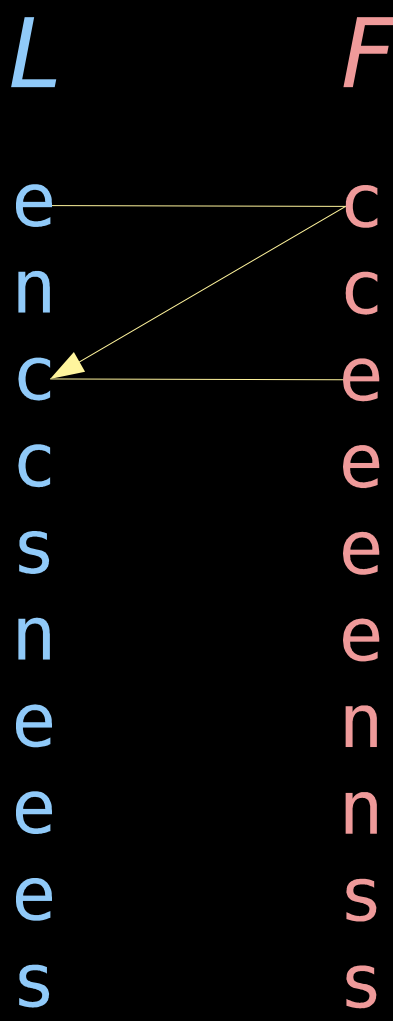
cycles



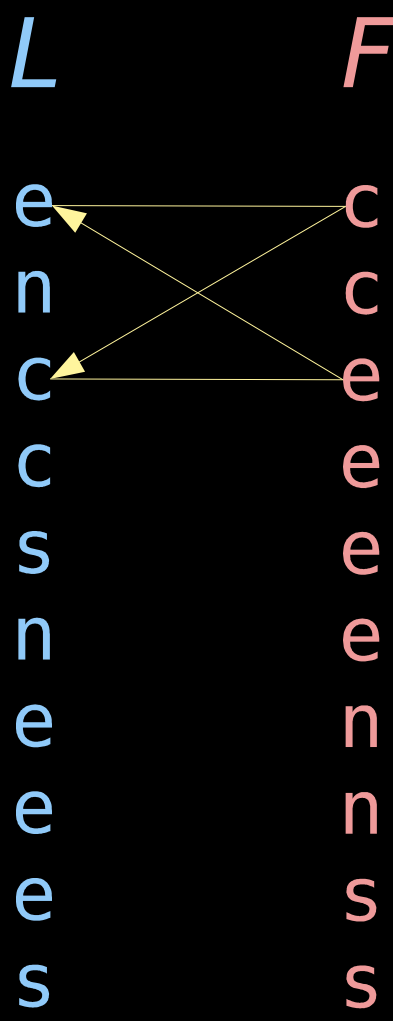
cycles



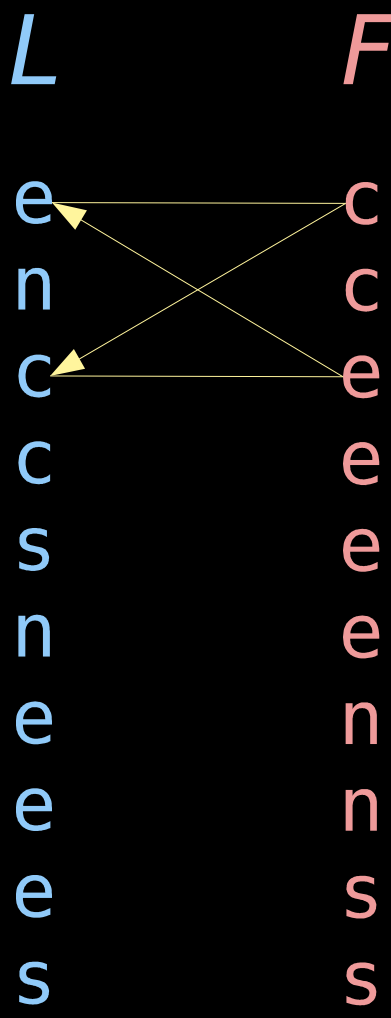
cycles



cycles

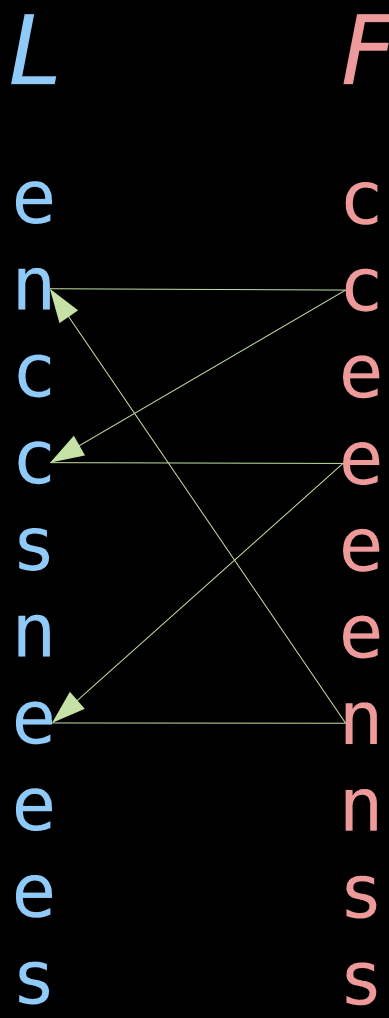


cycles



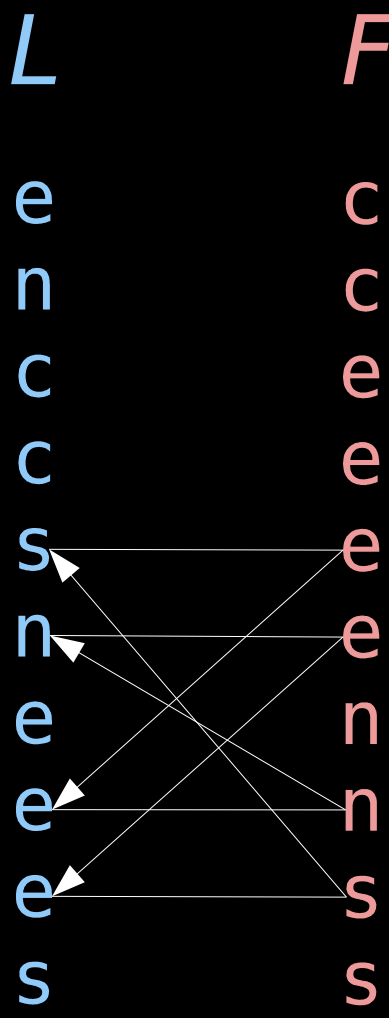
ce

cycles



ce

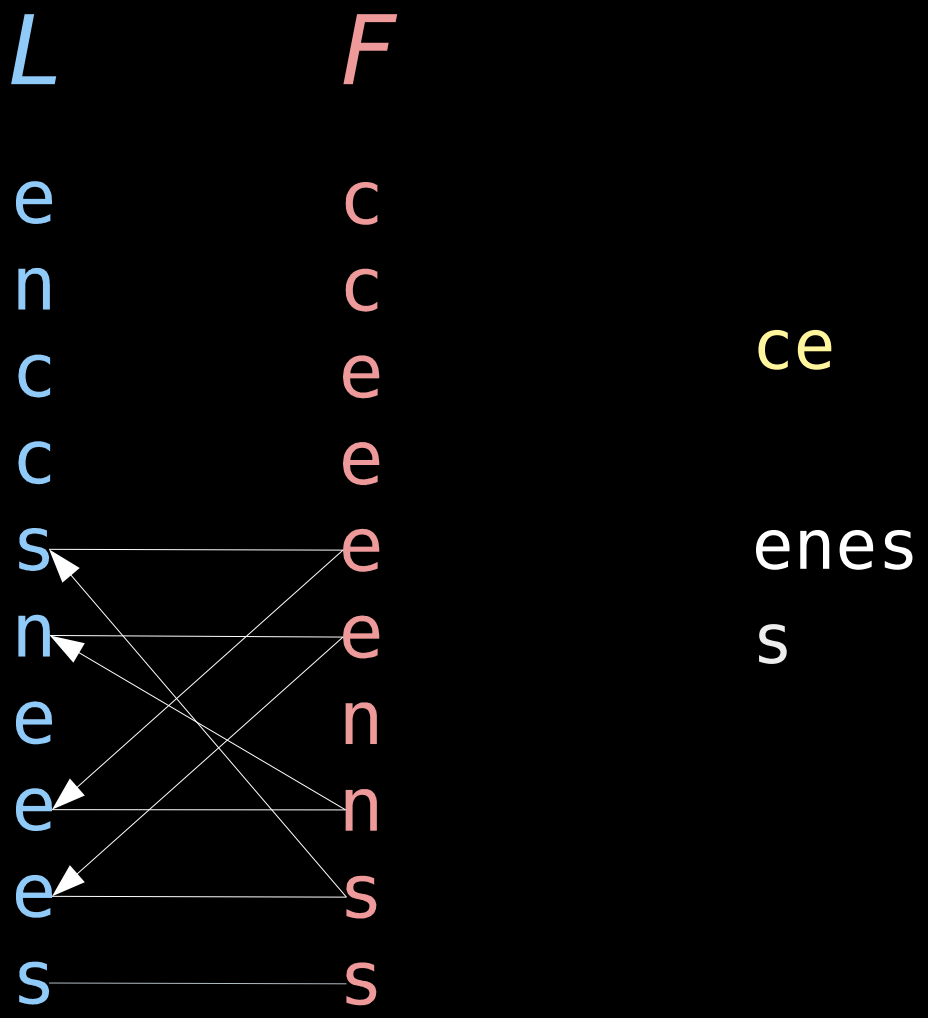
cycles



ce

enes

cycles

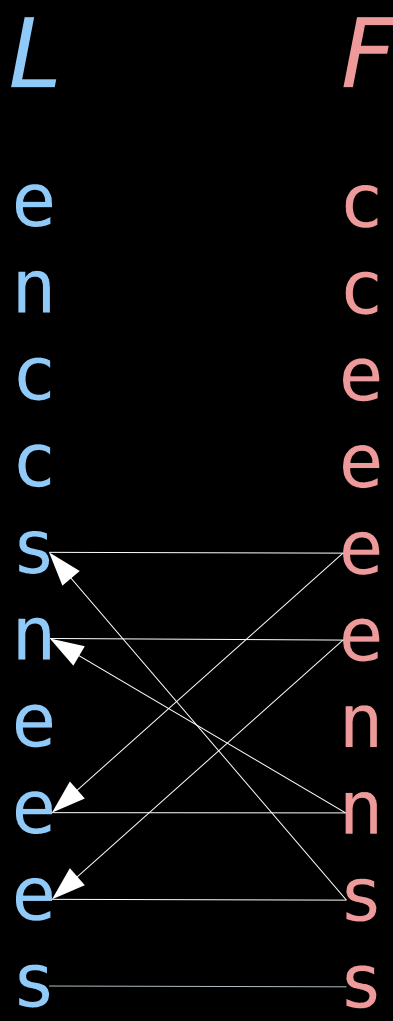


ce

enes

s

cycles


L
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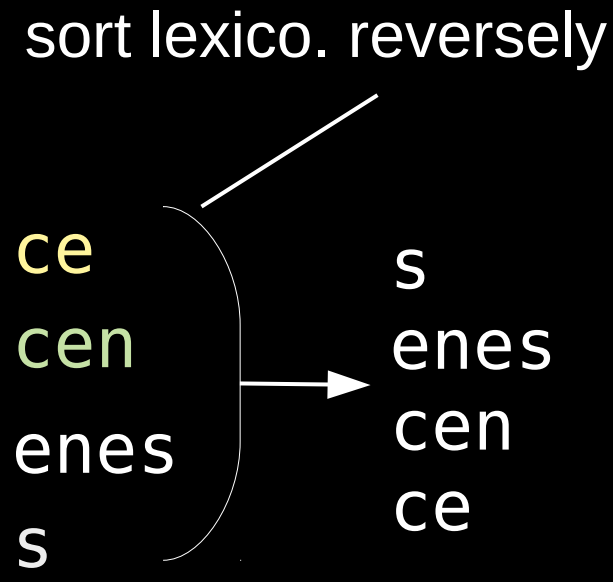
n
n
s
s


sort lexico. reversely
ce
cen
enes
s
s
enes
cen
ce

backward search 'cen'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

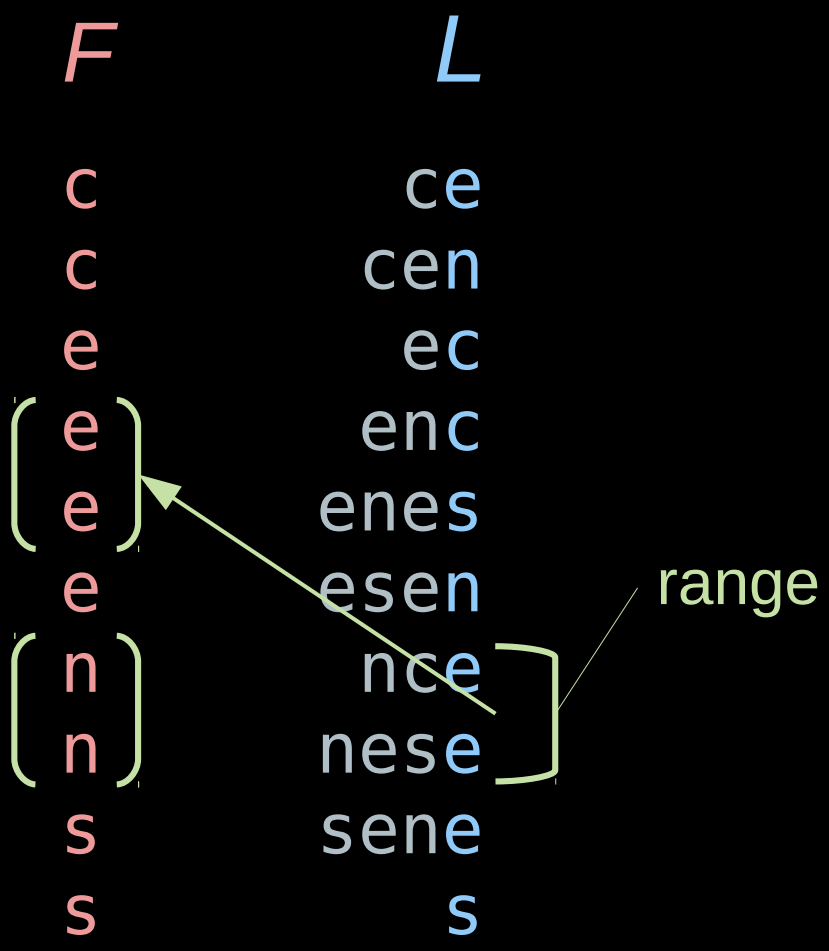
backward search 'cen'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

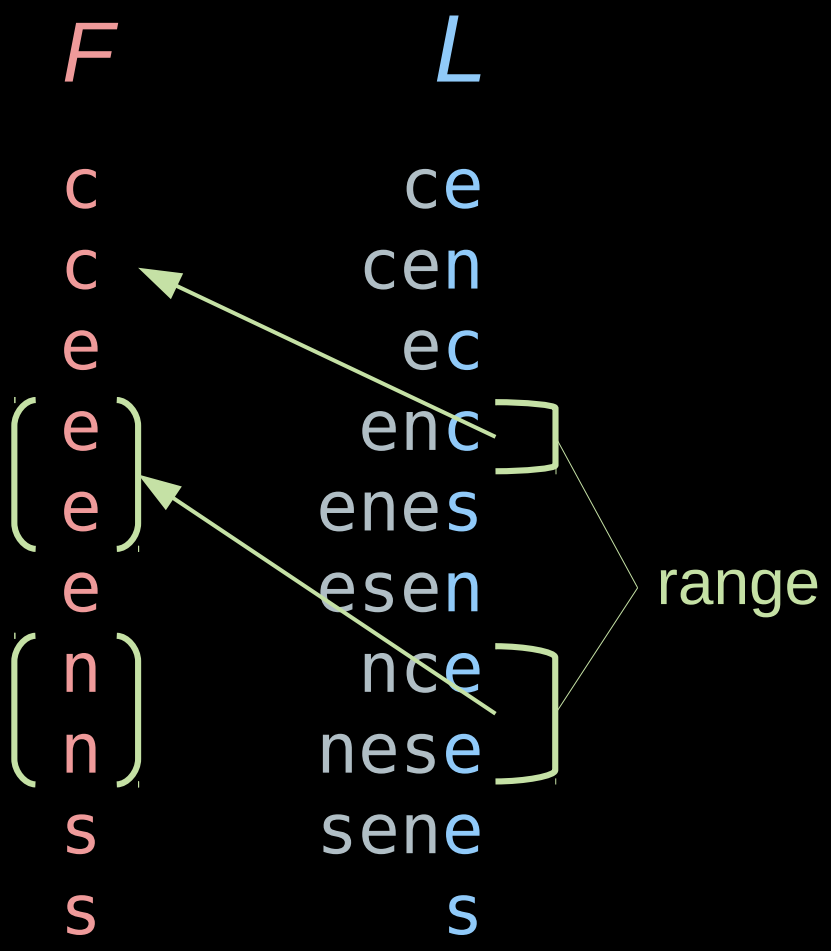
backward search 'cen'

s|enes|cen|ce

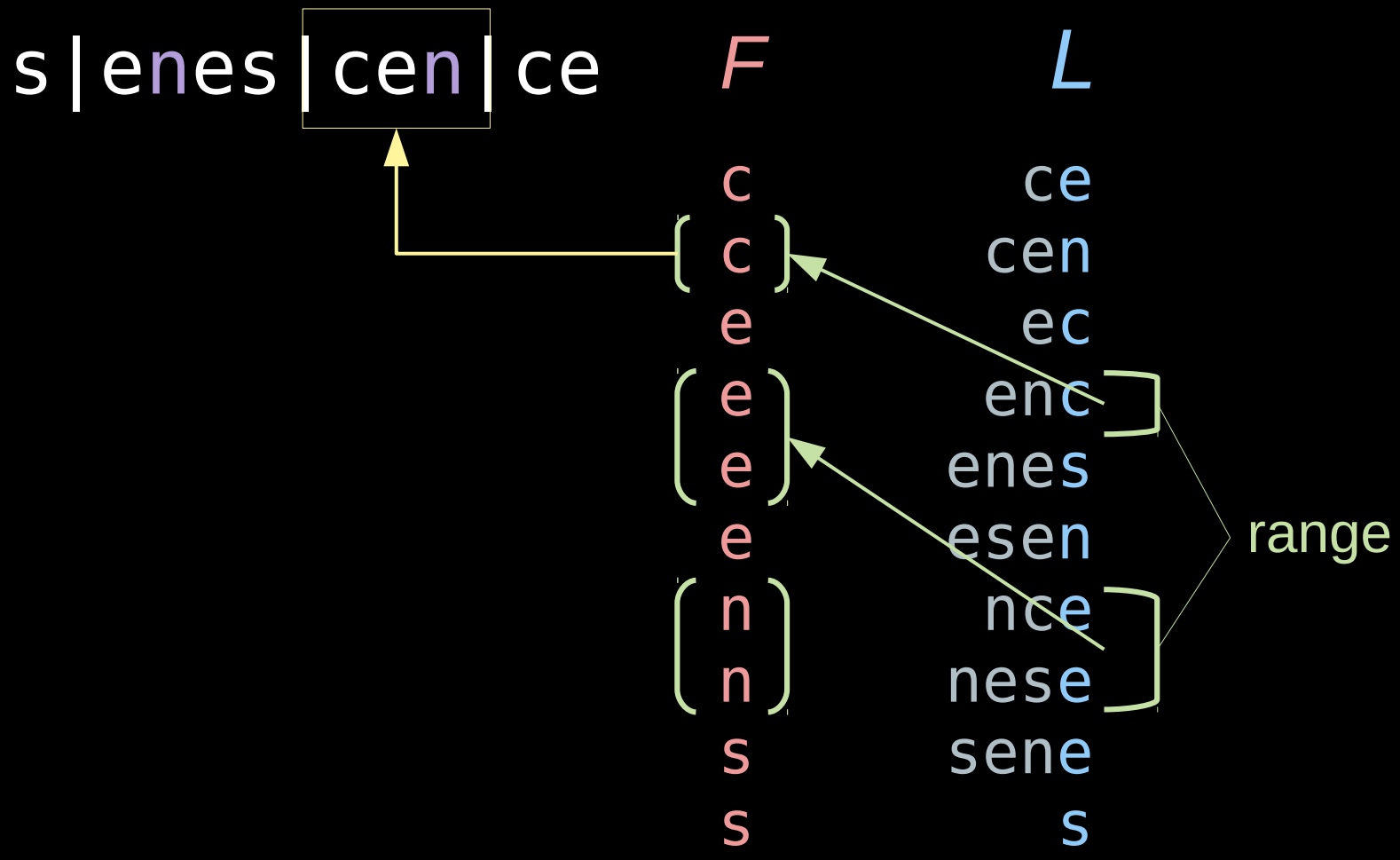


backward search 'cen'

s|enes|cen|ce



backward search 'cen'



backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

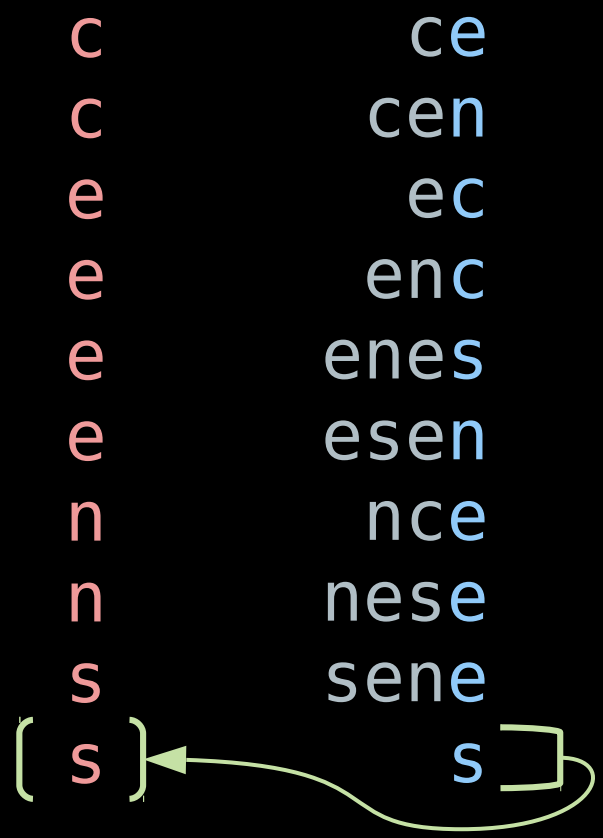
backward search 'ss'

s|enes|cen|ce

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

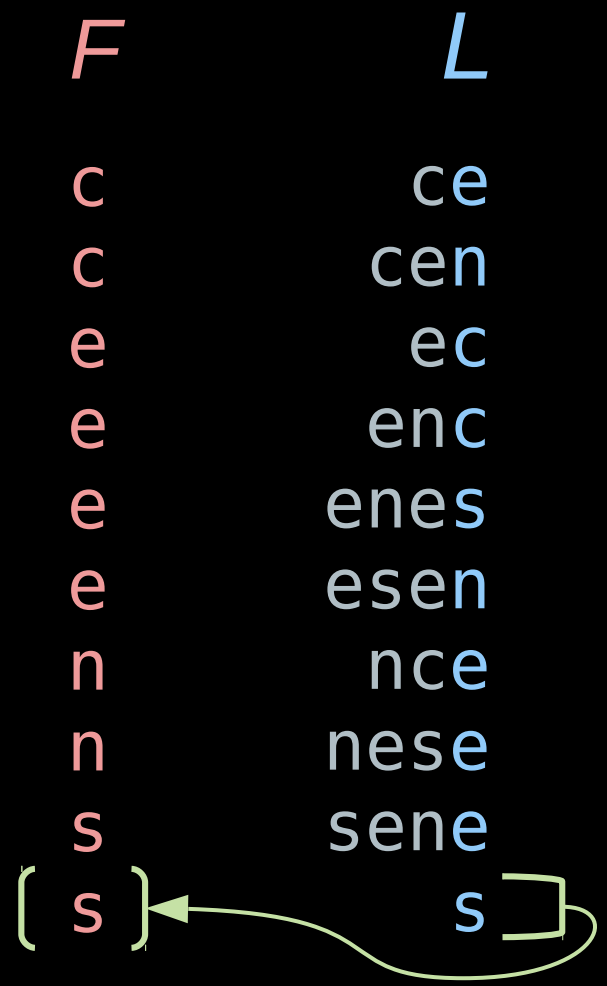
backward search 'ss'

s|enes|cen|ce



backward search 'ss'

s|enes|cen|ce



- cen is Lyndon word
- ss is **not**

pattern is Lyndon word

⇒ occurrences inside factors

⇒ found within cycles

backward search ≅ FM-index

pattern P is not a Lyndon word

- Lyndon factorization: $P = P_1 \cdots P_m$
- P_j substring of T_i or equal to T_i
- search P_m
- take care when starting with $P_{m\text{-}1}$!

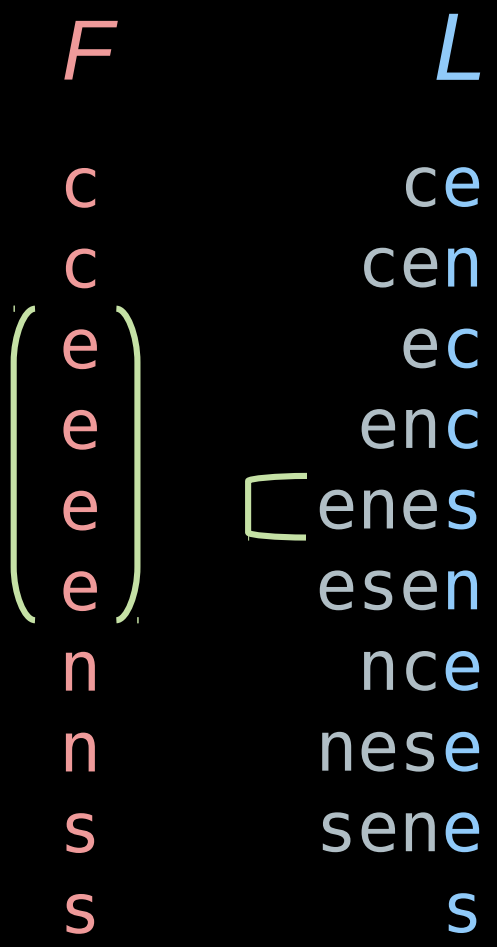
- backward search *P* = se

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

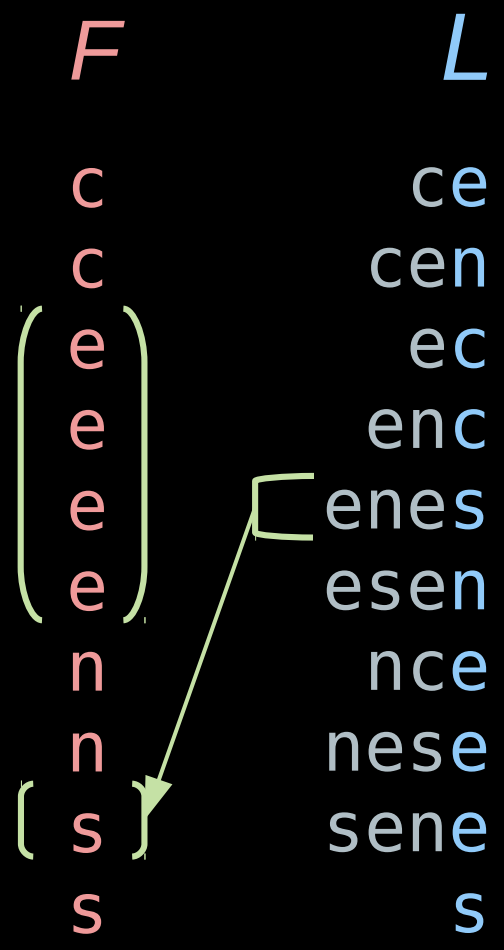
- backward search P = se
- P_2 = e

F	*L*
c	ce
c	cen
e	ec
e	enc
e	enes
e	esen
n	nce
n	nese
s	sene
s	s

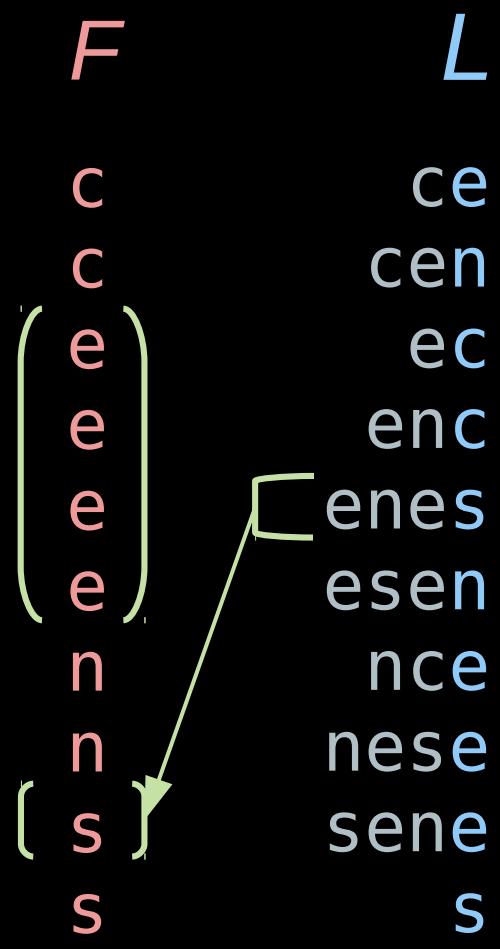
- backward search P = se
- P_2 = e



- backward search P = se
- P_2 = e
- P_1 = s



- backward search P = se
- P_2 = e
- P_1 = s

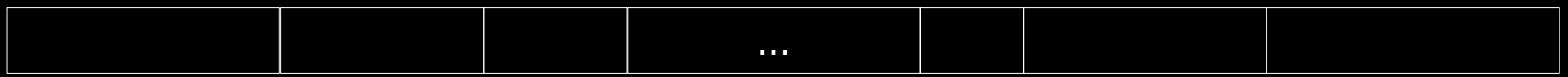


found

s|enes|cen|ce

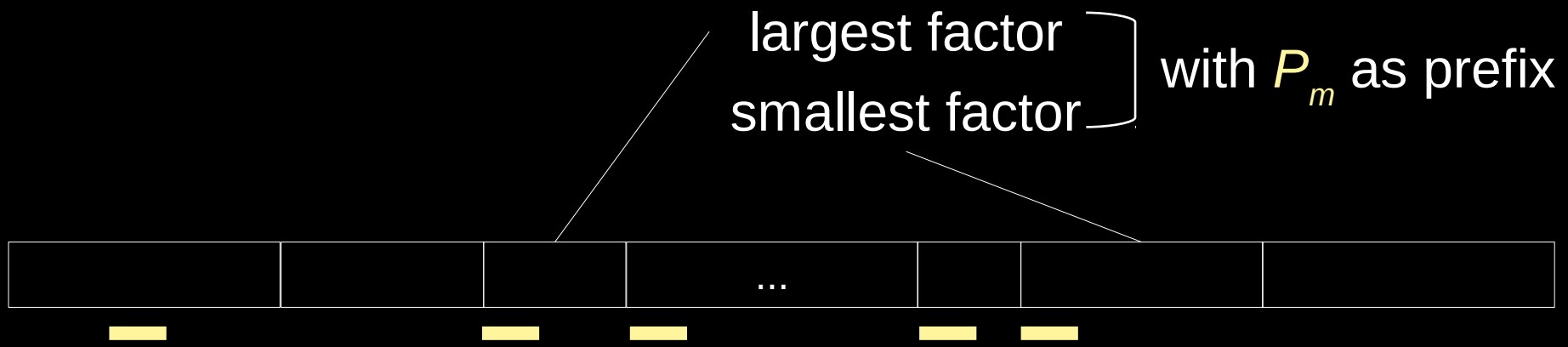
not counted

false occurrence

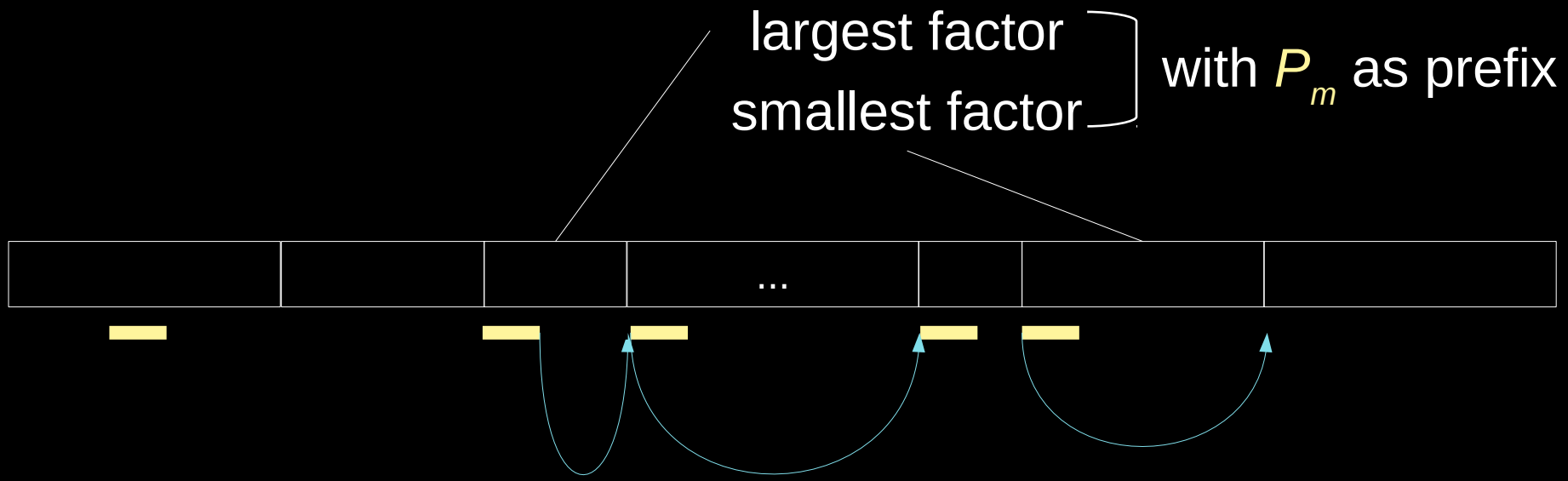


- backward search P_m

- backward search P_m



- backward search P_m
- continue search $P_{m-1} P_m$



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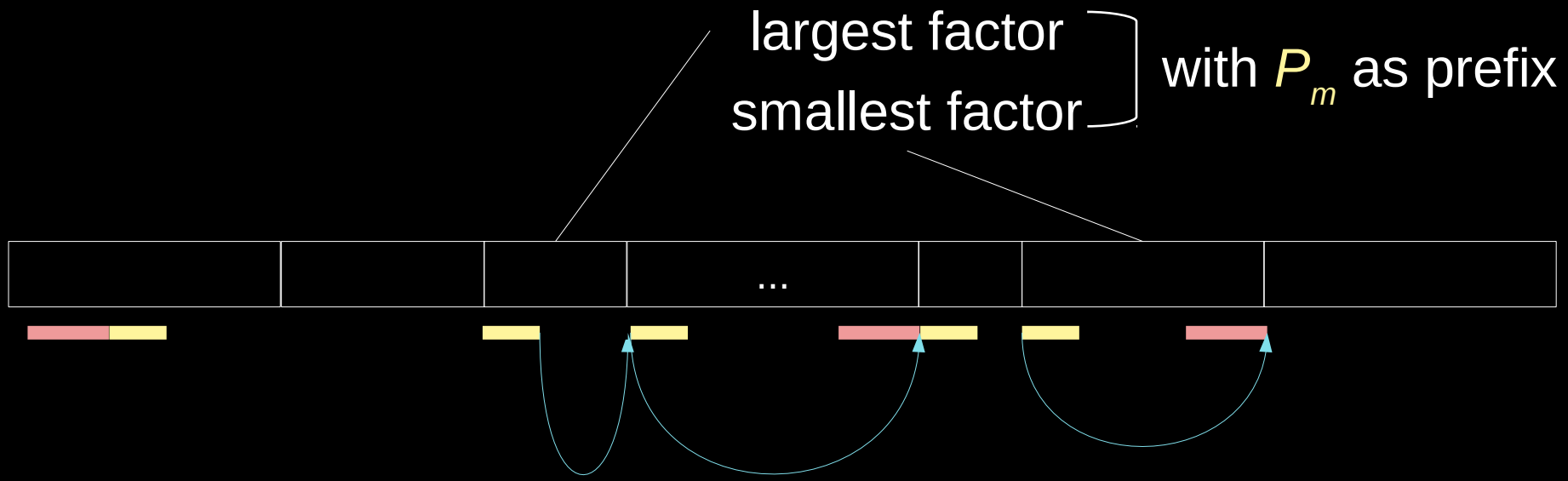

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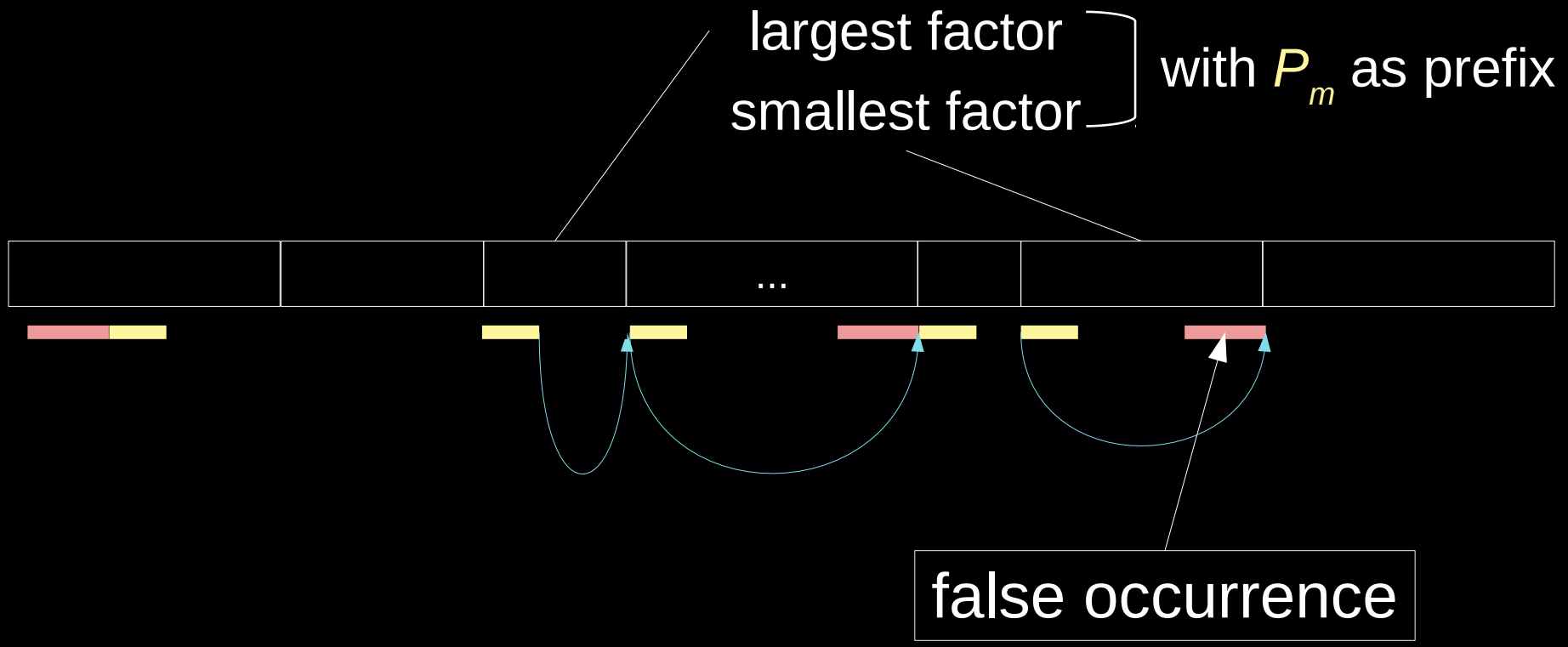

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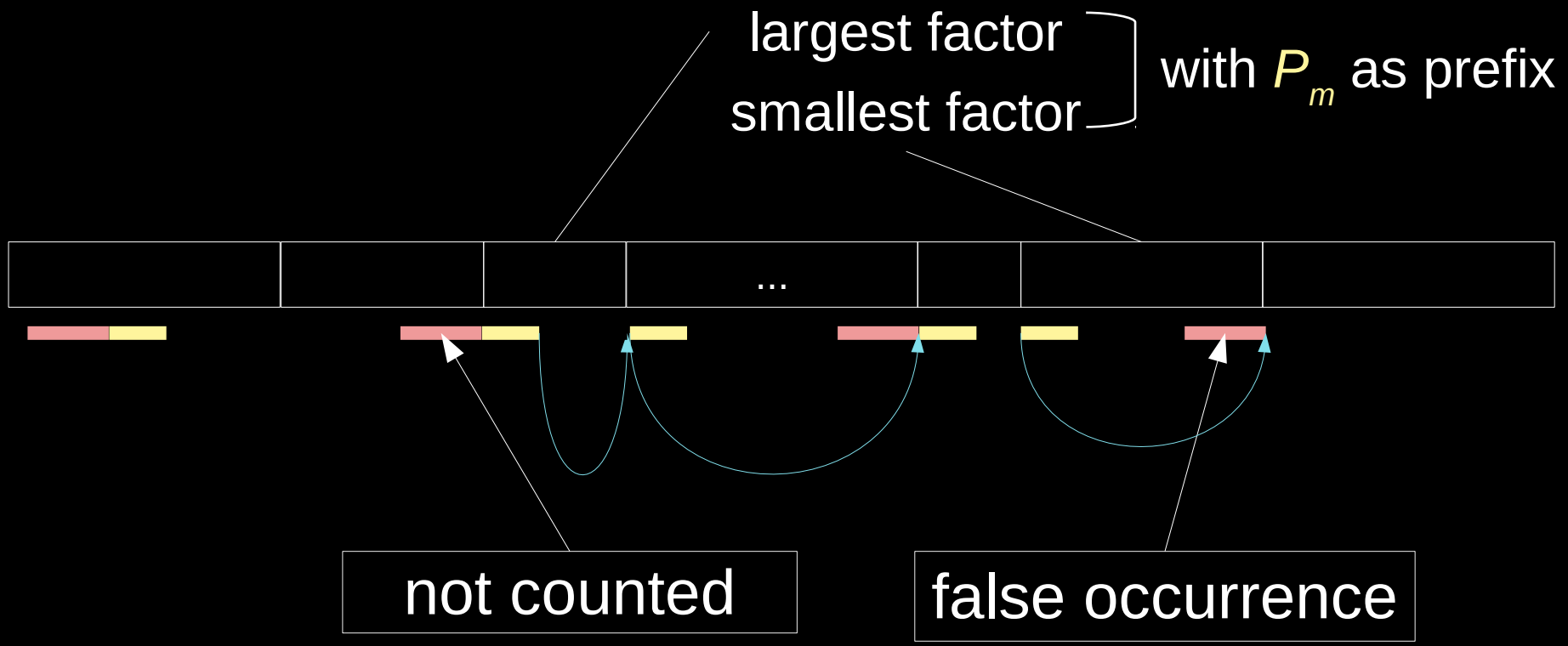
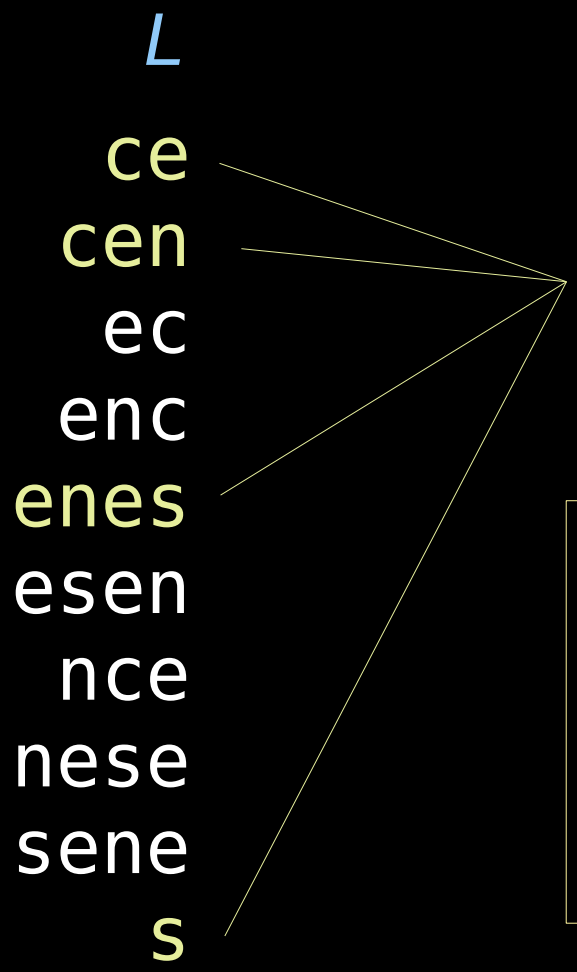


location of factors T_i



Lyndon factors $T_t \ldots T_1$

u, v Lyndon words:
$u <_{lex} v \iff u \prec_{\omega} v$

- after finding range of P_m :
  - for border $P_{m-1}P_m$ maintain
    - pointer to not-counted occurrence
    - pointer to false occurrence
- in total backward search on
  - range
  - at most $2m$ individual values

conclusion

- FM index with bijective BWT
- uses properties of Lyndon factorization on
  - text
  - pattern $P = P_1 \cdots P_m$
- currently O(m) times slower than FM index
- extended BWT does not seem to be a good candidate (no Lyndon word properties)

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